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PLUMB-LINE DEFLECTIONS AND GEOID IN EASTERN INDONESIA

AS DERIVED FROM GRAVITY

BY

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PREFACE.

The Netherlands Geodetic Commission publishes here a study made during the war by *Dr. J. E. baron de Vos van Steenwyk* about the shape of the geoid and the plumbline deflections in East Indonesia as derived from the gravity data in this area. For the first time the formulas expressing these relations have been applied here in practice and the results justify the great labour involved in their computation.

It may be of interest to mention the circumstances which gave Dr. De Vos van Steenwyk leisure to accomplish this great self-imposed task. Having been dismissed by the German authorities as Burgomaster of the town of Haarlem because of his loyal attitude towards the national cause, he deemed himself no longer safe at home and with reason; his house was searched three times for taking him prisoner. Dr. De Vos van Steenwyk disappeared and went into hiding for the four remaining years of the war. This time of inaction gave him a good opportunity for taking up again his scientific studies, for which his university career at Leiden, where he doctorated in Astronomy in 1918, had qualified him.

After the war Baron de Vos van Steenwyk resumed his official career. He is now Governor of the Province of North-Holland.

The map of this publication has been drafted by Dr. W. Nieuwenkamp, Professor of Geology at the university of Utrecht, and drawn by his assistant J. H. M. van Dijk.

F. A. Vening Meinesz,

President of the Netherlands
Geodetic Commission

PLUMB-LINE DEFLECTIONS AND GEOID IN EAST INDONESIA.

§ 1. In a communication of Jan. 28, 1928 to the Royal Academy of Sciences of Amsterdam, p.p. 315-331, Vening Meinesz deduced a formula for expressing the deflection of the plumbline as a function of gravity anomalies over the whole sphere.

This formula was found by differentiating the formula of Stokes expressing the distance N between the geoid and an adopted spheroid in the gravity anomalies over the whole earth, in a direction tangent to the geoid. For the validity of this formula, the conditions the chosen spheroid has to fulfill and the definition of the regulated geoid, obtained bij removing the outside masses, I refer to the above communication.

On page 322 he concludes:

„It appears doubtful if the accuracy of the result can ever „attain such a perfection that the formula could be used to „control the results of the triangulations by giving for each „astronomical station an equation for the latitude and lon- „gitude components of the plumbline deflections. Still, it gives „a useful control for traverse surveys, of which the errors are „so much greater, as is well known" . . . For this purpose, it „is of course necessary to have a sufficient number of gravity „stations in an area of somewhat greater extent than the area „of the survey".

The gravity determinations made by Vening Meinesz in Indonesia during the submarine expeditions in the years 1923-1930 and published in „Gravity expeditions at sea, Vol I and II" seem to offer an opportunity of testing this formula.

In 1941 Vening Meinesz mentioned this possibility to me and, as I had some time at my leisure, I offered to undertake these computations. The results are given in this paper.

For all stations the anomalies have been used after isostatic reduction according to the Hayford system; the deflections obtained are, therefore, not identical to the observed values but to the results found after a corresponding reduction for topography and isostatic compensation. As the anomalies have been computed with regard to the formula for normal gravity of Cassinis, the deflections have regard to the International Ellipsoid. Because of the way the formula of Stokes has been derived the center of this ellipsoid must be assumed to coïncide with the center of gravity of the Earth.

§ 2. THE OBSERVATIONAL DATA; AREA COVERED BY THE COMPUTATIONS; CHOICE OF THE STATIONS.

The East Indies are covered by a fairly dense net of gravity stations, all of which are located at sea or in harbours. In Dutch New Guinea it is supplemented by measurements taken at land stations for the geophysical survey of the island in the years 1934-1938: Considerable gaps exist consequently on the larger islands, especially Borneo, but, as we shall see, the regions where the deflections are so large as to be of interest are all east of Borneo.

The first computations were made along a profile roughly extending from Christmas island to Batavia. On this line the deflections at three landstations and five seastations were computed. It was chosen so as to cross at right angles the minimumline of gravity that runs south of Sumatra and Java.

The result of this trial (see the table on page 14) was that the plumbline deflections were considerable at the sea stations but that their values at the landstations were for all practical purposes negligible and even the most casual glance (confirmed by computation at two isolated points) at the gravity anomalies between Sumatra, Java and Borneo gives the certainty that plumbline deflections there would be still smaller and practically zero.

All efforts were therefore concentrated on the eastern part of the Archipelago, roughly speaking between the meridians of 119° E.L. and 134° E.L. The North and South boundaries are about $+5°$ and $-11°$, but the bulk of the points computed is situated between $+3°$ and $-9°$. It comprises the seas and islands situated on and about the trough separating the Asiatic continent from the Australian. Here gravity observations show great and abruptly changing anomalies and interesting results may be expected from the outset.

The choice of the points to be computed was a delicate one. If our aim should be to construct a chart of this whole region from which deflections could be read at each desired point, we had to cover it by a regular network of station with additional points where strong variations made it necessary to do so. If, on the other hand, we want an accurate value at a number of isolated points, e.g. at the astronomical stations of the hydrographic survey, we can dispense with the linking up of measurements. A third point of view is to choose the points where extreme values of the deflection may be expected, which means seeking out the points where the gradient of the gravity anomaly is strong.

The last two considerations have prevailed in our choice; the second condition has been fulfilled by choosing the astronomical stations of the hydrographic survey, with the exception of five which are unfavourably situated. The third condition is fulfilled by choosing stations on the profiles as published in Grav. Obs. a.S., Vol II.

An additional number of stations however was chosen to fill up the wide gaps left by the former and to control our assumption that in the mentioned parts the deflection will be negligible.

The actual choice was made in the following way: along the profiles, numbered from 2 to 12 (see table I) a number of 65 stations was chosen, some coïnciding with the gravity stations themselves, some others where the gradient of the gravity anomaly was steepest. Next to these, 33 stations, marked H, were added. These are all landstations, while the former were mostly, but not all, sea stations. When a H station occurred near a profile, it was substituted for one of that class. Lastly, a number of 30 additional stations, marked A, were chosen to fill up the gaps left. In this way a total of 136 stations was formed, of which 68 are at sea and 68 on land or in harbours.

§ 3. ADAPTATION OF THE FORMULA TO THE COMPUTATION.

The formula which gives the plumbline deflection as a function of the gravity anomalies over the whole earth reads as follows (publ. Acad. Amsterdam page 320).

$$\theta_x = \frac{1}{2\pi} \int_0^{2\pi} \cos \alpha \, d\alpha \int_0^{\pi} Q \, \Delta_0 \, d\psi \quad . \quad . \quad . \quad . \quad (1)$$

where

$$Q = \frac{\varrho''}{2\gamma} \cos^2 \tfrac{1}{2}\psi \left[\operatorname{cosec} \tfrac{1}{2}\psi + 12 \sin \tfrac{1}{2}\psi - 32 \sin^2 \tfrac{1}{2}\psi + \right.$$

$$\left. + \frac{3}{1+\sin \frac{1}{2}\psi} - 12 \sin^2 \tfrac{1}{2}\psi \times \lg \left\{ \sin \tfrac{1}{2}\psi \, (1 + \sin \tfrac{1}{2}\psi) \right\} \right]$$

The symbols used have the following meaning:

θ_x'' = the component along an X axis of the deflection

ϱ'' = cosec 1″

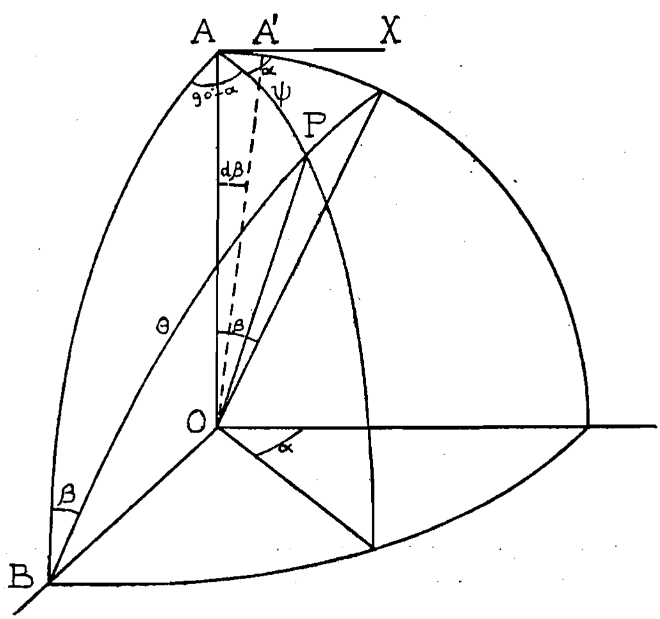
γ = the mean acceleration of gravity

α = the azimuth of a variable point P where the gravity anomaly is supposed known, measured from a direction AX (see fig. 1).

ψ = the distance AP; A being the point where the deflection is computed.

Δ_0 = the gravity anomaly corresponding to an element $d\sigma$ in P.

Q, which is ∞ for $\psi = 0$ decreases rapidly and at a distance of 10°, is only 1.59.



(Fig. 1)

As stated in the above quoted paper, for small ψ we can neglect in Q all terms after the first, and, by introducing instead of ψ the linear distance $AP = r$, so that $r = 2\,R \sin \frac{1}{2}\,\psi$ where R stands for the mean radius of the earth, we get:

$$Q = \frac{1338}{r} \quad (r \text{ expressed in km})$$

By substituting in (1) the surface element df in stead of $d\,\psi$ and $d\,\alpha$, we find for the effect of the anomalies in the neighbouring region:

$$\theta_x'' = \frac{\varrho''}{2\,\pi\,\gamma} \int \cos\alpha \, \frac{\Delta_0}{r^2}\, df \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Here it appears that the plumbline deflection reacts in first approximation as it would do if each surface element df contained a mass $\frac{\varrho''}{2\,\pi\,\gamma}\,\Delta_0 df$ which was attracted by the point A according to the law of Newton with a coefficient equal to the unity.

Theoretically the integrations in (1) should be extended over the whole sphere. Practically, they can be limited to the region where the gravity was determined. This will be allowed, as the influence diminishes as the inverse square of the distance and the values of Δ_0 will cancel each other for the major part in the outer rings. For practical reasons we have limited the field to a zone with a radius of about 5° (555 km.).

Our next purpose shall be to ascertain if for such a field the simplified formula (2) can be used. If so, computations will be shortened very materially. For if we divide the field, firstly into zones by circles, the radii of which will grow in a fixed proportion. the total factor by which the mean Δ_0 of each zone has to be multiplied will be a constant. By subdividing each zone into compartments by radii that make angles with the X-axis of which the sines have a constant difference, the influence of the factor $\cos\alpha$ under the $\int$ is taken into account, and, having adopted for each compartment a mean value for Δ_0, we can simply add those values, taking care to change the sign in two quadrants. After having found θ_x'' in this way, a second computation has to be made for θ_y''.

A simple inspection of formula (1) shows however, that if we require an accuracy of ½ % in Q the formula (1) will not stand being simplified into (2) up to a distance of $\psi = 5°$. For this limit we get:

$$\begin{aligned}
+\operatorname{cosec}\tfrac{1}{2}\psi &= +\,22.926 \\
+\,12\sin\tfrac{1}{2}\psi &= +\quad 0.523 \\
-\,32\sin^2\tfrac{1}{2}\psi &= -\quad 0.061 \\
\frac{3}{1+\sin\tfrac{1}{2}\psi} &= +\quad 2.875 \\
-\,12\sin^2\tfrac{1}{2}\psi\log\left[\sin\tfrac{1}{2}\psi\left(1+\sin\tfrac{1}{2}\psi\right)\right] &= +\;0.070.
\end{aligned}$$

The deviation of the first term by the third and fifth terms can be neglected, but not by the second and the fourth.

If we wish to retain the advantage of dividing the field into compartments where the multiplying factor of Δ_0 shall be equal, we have to use a stratagem.

It consists in modifying the width of the successive rings in such a way that the deviations of the value of Q, according to (1), from the simplified form $Q = \frac{1338}{r}$ shall be compensated by a corresponding variation of the factor which links up the radius of each zone with the next one. The way in which this has been done will now be set forth in detail.

The first radius r_0 was taken at 30 km. The central field within this ring will be investigated separately (see p. 13). The next radius r_1 was put at 45 km. a.s.f. till $r_7 = 512.5$, yielding seven zones in total.

The function Q (including the third and fifth term) was tabulated with intervals of 5′ of the argument ψ up to 5°. For the purpose of facilitating the interpolation, the inverse function 1/Q was also tabulated.

In order to be able to compare how Q decreases in the successive zones, a mean radius ϱ has to be taken in each zone.

Now, the mean value ϱ for all the surface elements within a ring will be the radius which divides the zone into two equal parts, consequently

$$r_n^2 - \varrho_n^2 = \varrho_n^2 - r^2_{n-1} \quad \text{or} \quad \varrho_n^2 = \frac{r_n^2 + r^2_{n-1}}{2}$$

and, because $r_n = \frac{3}{2} r_{n-1}$, $\varrho_n = 1{,}275\ r_{n-1}$.

Expressing the values for ϱ in arc we get the following table:

	Q	S
$\varrho_1 =$ 20′.7	0.02829	0.02829
$\varrho_2 =$ 31′.0	0.04226	0.04244
$\varrho_3 =$ 46′.5	0.06285	0.06365
$\varrho_4 =$ 1° 9′.9	0.09347	0.09547
$\varrho_5 =$ 1° 44′.7	0.13783	0.14321
$\varrho_6 =$ 2° 37′.0	0.20186	0.21482
$\varrho_7 =$ 3° 55′.6	0.29216	0.32223

The values in the second column can be interpolated from the table for $1/Q$. The values S in the third column have been found by starting from $S_1 = Q_1$ and multiplying by 3/2.

If now we want to diminish the surface of a zone in the proportion $\frac{S_n}{Q_n}$ which we may put as $\frac{1}{1+\varepsilon_n}$, where ε_n is a small quantity, we have to replace the outer radius r_n by a radius x_n where

$$(x_n^2 - r^2_{n-1})\,(1+\varepsilon_n) = r_n^2 - r^2_{n-1}$$

which gives
$$x_n^2 = \frac{r_n^2 + \varepsilon_n r^2_{n-1}}{1+\varepsilon_n}$$

which expression (because ε is small) reduces to:

$$x_n = (1 - \tfrac{1}{2}\varepsilon_n)\left(1 + \tfrac{1}{2}\varepsilon_n \frac{r^2_{n-1}}{r^2_n}\right) r_n$$

Substituting $r_{n-1} = \frac{2}{3} r_n$ we get finally:

$$x_n = (1 - \tfrac{1}{2}\varepsilon_n)(1 + {}^2/_9\,\varepsilon_n)\, r_n.$$

Starting from the first zone, with boundaries $r_0 = 30$ km. and $r_1 = 45$ km. we can now build up successively a set of corrected zones:

$\varepsilon_2 = 0.004$	$x_2 = 67.4$ K.M.
$\varepsilon_3 = 0.013$	$x_3 = 100.6$,,
$\varepsilon_4 = 0.022$	$x_4 = 150.1$,,
$\varepsilon_5 = 0.038$	$x_5 = 222.6$,,
$\varepsilon_6 = 0.062$	$x_6 = 328.1$,,
$\varepsilon_7 = 0.103$	$x_7 = 477.3$,,

Having in this way obtained a division into zones, we next subdivide the quadrants of each zone into four parts, according to the way set forth above. The radii are making angles with the X axis successively equal to arcsin 1/4, arcsin $\frac{1}{2}$ and arcsin 3/4.

The whole field under consideration is now divided into 112 compartments, for each of which the corresponding value of Δ_0 has to be multiplied by the same factor (presently to be ascertained) and a central field, which requires separate consideration.

§ 4. THE INTERPOLATION OF Δ_0.

Inside each compartment a value for Δ_0 has to be adopted, derived from the measured gravity anomalies, scattered over the field. Of course Δ_0 will vary also within each compartment and a choice has to be made of a point that is to be considered as representing the whole area of that compartment.

It was deemed advisable to choose those points on the radii which make with the X axis the angles arcsin 1/8, 3/8, 5/8, and 7/8 respectively, and at distances $y_n = \sqrt{x_n x_{n-1}}$.

These points having been chosen, the framework of the circles and radii can be dispensed with and the diagram actually used in the computations contained only those 112 points (with the corresponding radii for sake of identification).

Of this diagram (see fig. 2) constructed on the scale 1 : 3.10^6 (1 c.M. = 30 km.) a clichè was made and a thousand copies printed on transparent paper.

For each station two diagrams had to be used, one for θ_x'' (deflection in longitude) and one for $\Delta\,\theta_y''$ (deflection in latitude). On each diagram the observed values of Δ_0 were put in (about 30 to 50 in number) and the values for the 112 points interpolated. This inter-

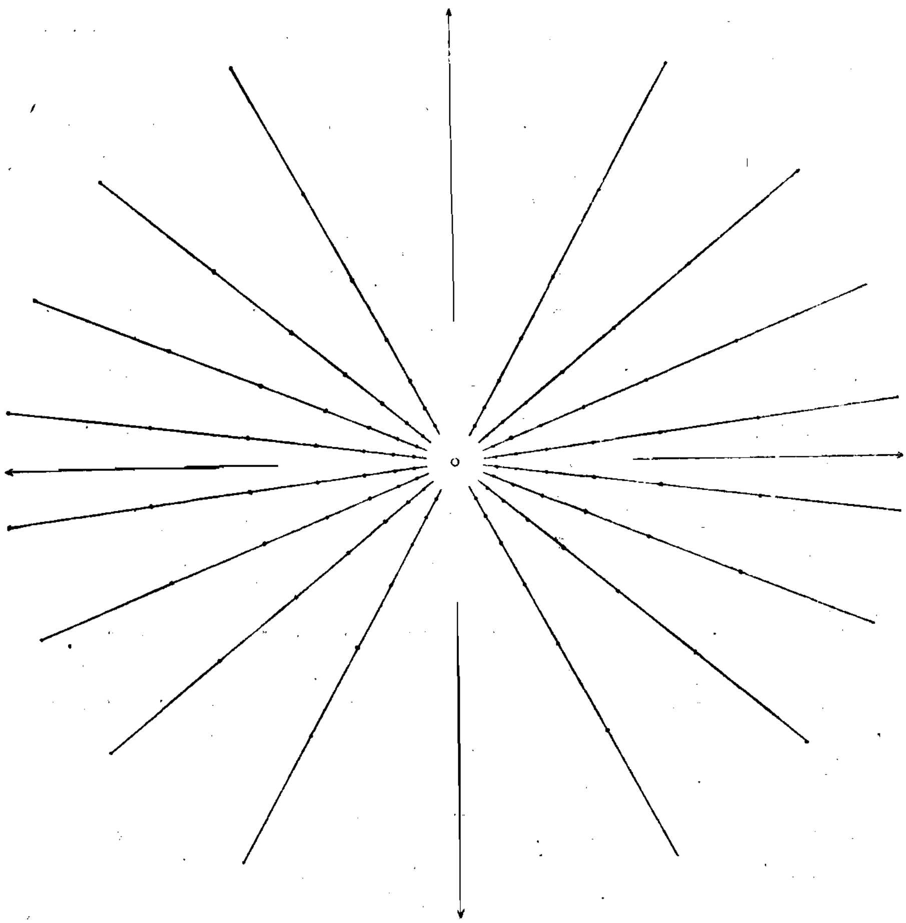


Fig 2

polation constitutes the most delicate part of the reduction. There are few indications as to which method of interpolation will best represent the actual conditions. The profiles published on page 87 of Gravity Observations at sea Vol. II have been extensively used by inserting interpolated values from them in the diagram, but outside these Δ_o shows often abrupt changes which leave a certain amount of arbitraryness in the choice of intervening points. When all other indications failed, linear interpolation had to be used. The problem can be compared to the task of finding the height of a certain number of spots in a mountainous region, say Switzerland, when disposing only of a limited number of measurements, scattered haphazardly over the country, and a very hazy idea of the general features of the country. We will revert to this problem when we shall discuss the accuracy of our results.

The values of Δ_o found in this way were simply added together, taking care to change the sign in the second and third quadrant, and this sum had to be multiplied by a factor deduced as follows:

Reverting to formula (1)

$$\theta_x'' = \frac{1}{2\pi} \int_0^{2\pi} \cos a \, d\, a \int_0^{\pi} Q \; \Delta_o \; d\, \psi$$

we can now discard for Q all but the first term and put $\cos^2 \frac{1}{2} \psi = 1$

or $Q = \dfrac{\varrho''}{2\gamma} \times \dfrac{2R}{r}$ when R is the mean radius of the earth.

(r distance along a great circle from the centre to the point for which Δ_o was interpolated).

Substituting $d\psi = \dfrac{dr}{r}$ formula (1) reduces to:

$$\theta_x'' = \frac{\varrho''}{2\pi\gamma} \int_0^{2\pi} \cos a \, d\, a \int_{r_0}^{r_n} \Delta_o \frac{dr}{r}$$

the second integral covering the field used outside the central part.

For one compartment, comprised between the angles α_1 and α_2 and the radii r_1 and r_2 we have adopted a constant value for Δ_o

giving: $$\Delta_o \int_{r_1}^{r_2} \frac{dr}{r} = \Delta_o \ln \frac{r_2}{r_1}$$

and consequently: $$\theta_x'' = \frac{\varrho''}{2\pi\gamma} (\sin \alpha_2 - \sin \alpha_1) \ln \frac{r_2}{r_1} \times \Delta_o$$

Substituting the numerical values for a and r

$$\theta_x'' = \frac{\varrho''}{2\pi\gamma} \times \tfrac{1}{4} \ln 1.5 \; \Delta_o$$

and putting $\varrho'' = 206265$ and $\gamma = 979.77$ (mean value over the earth according to the formula of Cassinis for normal gravity) we get ultimately:

$$\theta_x'' = 0.0034025\ \Delta_o.$$

For the central field (diameter 60 km.) we must suppose that Δ_o has a constant gradiënt

$$\Delta_o = c + \beta x$$

The first term doesn 't contribute to θ_x'' and may consequently be discarded.

Putting $x = r \cos \alpha$ formula (1) reduces to

$$\theta_x'' = \frac{c''\beta}{2\pi\gamma} \int_0^{2\pi} \cos^2 \alpha \, d\alpha \int_0^{r_0} dr$$

Now $\int_0^{2\pi} \cos^2 \alpha \, d\alpha = \pi$ so $\theta_x'' = \frac{\varrho''}{2\gamma} \beta r_o$

If we denote the difference between Δ_o at both extremities of a diameter $2 r_o$ as v_o this means $v_o = 2 \beta r_o$ and

$$\theta_x'' = \frac{\varrho''}{4\gamma} v_o = 0.0527\, v_o.$$

As will appear from our discussion of the results, the contribution of the central field will only be a smal fraction of the whole, thus justifying the choice of the inner ring at 30 km.

§ 5. PRELIMINARY COMPUTATIONS.

As stated above, a first test of the formula was made in a profile from Christmas island to Batavia and a few points in that neighbourhood. The results are given below.

The deflections are counted positive to the East and the North, according to the ordinary geodetic conventions. The azimuth φ is thus reckoned from East over N-W-S.

For the points where θ is large we have checked the computations in the following way. Adopting the angle φ as found by the first computation, the diagram was rotated through this angle and its principal axis made to coïncide with the direction of the total deflection. In this position a new determination was made of θ, a single diagram now only being needed, as the deflection at right angles should be zero. A small error in φ will not materially affect the value of θ and, besides getting a control on the accuracy of our interpolation, we eliminate to a certain extent the variation of Δ_o inside each compartment. These control values are given as θ_o

STATION	β	γ	θ_x	θ_y	θ	φ	θ_c	Distance (in km)
Priok	$-6^\circ\ 0'$	$107^\circ\ 0'$	$+1''.4$	$-1''.0$	$1''.7$	325°		} 69
Buitenzorg	$-6^\circ\ 36'$	$106^\circ\ 49'$	$+2''.9$	$-1''.3$	$3''.2$	336°		} 49
	$-7^\circ\ 0'$	$107^\circ\ 0'$	$+2''.5$	$+0''.8$	$2''.6$	18°		} 65
Sea	$-7^\circ\ 35'$	$106^\circ\ 55'$	$+5''.1$	$+11''.5$	$12''.6$	66°	$11''.7$	} 61
	$-8^\circ\ 5'$	$106^\circ\ 42'$	$+7''.7$	$+22''.0$	$23''.3$	71°	$23''.4$	} 68
	$-8^\circ\ 40'$	$106^\circ\ 31'$	$-2''.1$	$-3''.3$	$3''.9$	238°	$3''.2$	} 54
	$-9^\circ\ 8'$	$106^\circ\ 24'$	$-9''.4$	$-23''.7$	$25''.5$	248°	$26''.1$	}106
	$-10^\circ\ 0'$	$106^\circ\ 0'$	$-3''.3$	$-7''.7$	$8''.4$	247°	$8''.6$	

ISOLATED STATIONS

Lembang	$-6^\circ.50'$	$107^\circ.37'$	$-0''.1$	$-0''.3$	$0''.3$	
Sea	$-8^\circ\ 0'$	$108^\circ\ 0'$	$+4''.5$	$+15''.2$	$15''.8$	$73^\circ.5$
Sea	$-8^\circ\ 0'$	$104^\circ\ 0'$	$-10''.2$	$-16''.3$	$19''.2$	238°

in the last column but one. The last column gives the linear distances between two successive stations and is very illustrative of the rapid changes in θ.

The θ_c are in very satisfactory agreement with the original ones, but, as will appear lateron, this is not always the case.

The values of θ_x and θ_y have been represented on a graph and it appeared that a smooth line can be drawn through them, which makes an interpolation for intermediate stations rather easy.

The two other conclusions that can be drawn from this preliminary computation are, firstly that in the regions where the gravity anomalies are rapidly chancing the deflection of the plumbline may also vary to an appreciable amount over distances of some ten km. Secondly that for Java, and the same will hold for Sumatra, the deflection is small. This latter fact has determined the choice of the other stations in the eastern part of the Archipelago.

§ 6. DISCUSSION OF THE RESULTS FOR THE EASTERN ARCHIPELAGO.

An inspection of the results as tabulated on p. 19 e.s. shows that the values of θ range from zero to 40″.3. These values are distributed as follows over the 136 stations.

0″— 5″	35	20″—25″	7
5″—10″	45	25″—30″	5
10″—15″	18	30″—35″	2
15″—20″	18	35″—40″.3	6

Now this distribution is not representative for the area as a whole, because the choice of the stations has been made so as to favour extreme values of θ.

For the major part of this area, although it shows the greatest anomalies of the archipelago and, what matters here, the steepest gradients of this quantity, the value for θ will remain below say 5 seconds of arc. It is only in a rather well-defined belt, on each side of the sweeping curve of the minimum-line of gravity anomalies that appreciable figures appear. The influence of remoter regions also has a smoothing character on the top values.

The deflection θ, as we have explained before, is composed of two parts, one from the central field, the other from the rings around it. The table gives only the total values, but it is of interest to note that the part due to the central field is always the smaller of the two. In no case it exceeds 10″ and it averages 20 % of the whole deflection.

This is important to keep in mind when we try to get an insight into the mean error of θ. This mean error is due to the following causes. First to the mean error of Δ_o, the gravity anomaly itself, which we may put down as 4 mgal. Its influence on the central field, where the difference v_o (see page 13) has been *the difference of the mean of two values of* Δ_o, on each extremity of its diameter, multiplied by a factor 0.0527 is consequentily 0.0527 ε_o where ε_o stands for the mean error of Δ_o. Its influence on the rings, where 112 interpolated values for Δ_o are added and then multiplied by 0.0034 is 0.0034 ε_o $\sqrt{112} = 0.036\,\varepsilon_o$.

Next to this source of error, there are two other ones: the interpolation error and the assumption that inside each compartment the value of Δ_o remains constant. These two factors contribute certainly more to the total mean error than the first one, but are difficult to evaluate. They are greatest for those stations where Δ_o is rapidly changing in many compartments, which roughly speaking goes with large values of θ. An estimate a posteriori can be made in those cases where a second computation was made after rotating the diagram through a certain angle. The differences found here must be wholly ascribed to the last two sources. As these stations were all chosen on account of their large values for θ, and because of the difficulties experienced in interpolating Δ_o, their accuracy will compare unfavourably with the rest. The deviations from the mean of the two values found for θ averages at 31 stations (marked C in the table) 0″.7, which is rather less satisfactory than the results of the prelimary computations (see page 14).

This we can ascribe to the more complicated pattern of Δ_o in the eastern archipelago, compared with the straight profile Christmas Island-Batavia. When the minimum line of Δ_o curves or branches off it renders the interpolation much more difficult. Inside the triangle of minimum values between Celebes and Halmaheira it was even found impossible to locate a station that could furnish reliable information of the value for θ at such a spot. Happily we can be certain that on such points θ will have a small value anyhow.

From our experience in this work of interpolating we may conclude that it would be useless to try to assign a mean error to the values of θ. Conditions vary too much from one station to another As a rule, small values will be more accurate in absolute measure than large ones, but I feel confident that in the larger values the error will be nowhere in excess of 10 %: Apart from the stations labelled c, which rest on a double determination, we have marked some stations *„strong"* or *„weak"* as we found the interpolation exceptionally easy or difficult.

The stations on New-Guinea offer a peculiarity as here the central part and inner rings are very strongly determined, but for the eastern parts of the outer rings Δ_0 is wanting completely. Still we may consider them among the better determinations. In the southern part of our area, Δ_0 is also completely wanting on the Sahoul Shelf, but here there seems reason to assume that it will be practically zero, which might not be the case in the Pacific Ocean east of New Guinea.

As to the accuracy of φ, it naturally depends primarily on the absolute value of θ. For the largest of its values, it will be accurate to a few degrees, for $\theta < 5''$ it becomes largely uncertain.

§ 7. THE DRAFTING OF THE GEOID-MAP.

The results of our computations as given in the table can be visualised in the following way.

The plumbline deflection θ being in fact the gradient of the distance N between the geoid and the spheroid, in the direction determined by the angle φ, or $\theta = dN/ds$, we can draw at each station a line orthogonal to φ which is the tangent in that point to the direction of an isoline for N and we thus obtain, therefore, a map of isohypses for the elevation N of the geoid with regard to the ellipsoid. Those lines can be drafted where the stations are sufficiently densely distributed over the map. But the linking of the different parts of those isohypses is hampered by the fact that we only know dN/ds and not N itself. This gap in our knowledge can be partially filled up by computing values of N itself by means of the theorem of *Stokes*. This has been done for 21 stations, adequately chosen, which are listed below in Table II; they are indicated in the map.

We shall not enlarge here on the method of computing N; this computation did not offer any special difficulties. The field of gravity anomalies from which these values have been derived had to be identical to that used for determining the plumbline deflections and so this field was limited in the same way to the anomalies in the Archipelago as published in „Gravity Expeditions at Sea", Vols I and II; the values of the anomalies were again used after the isostatic reduction according to the method of Hayford. For all the areas outside the archipelago no anomalies have been introduced and so the picture of the geoid given by the map has to be supplemented by the values of N given for this area by the anomalies over the whole remaining part of the Earth. It is likely that this as yet unknown part of N will only show slow changes over the area of the map.

Our map shows great irregularities; we find differences in N of some 20 meters at distances of only a few hundred kilometers. These irregularities are of course brought about by the great deviations from the normal mass-distribution in this area as revealed by the large size of the gravity anomalies.

Besides using the direction φ in each point where the plumbline deflection has been computed and besides making use of the values of N of Table II. we can also profit for the drawing of the isohypses of our knowledge of the absolute value of θ at each staton. This provides us with the density of those isohypses and this gives us a further guide for the drafting of the curves.

Of course we can also use our isohypses conversely for reading, with more or less confidence, at each point of the map the direction and the magnitude of the plumbline deflection, which is the aim of this research. The scale of the distance between the isohypses has been chosen in such a way that it corresponds to a difference of one meter in N. As the map is on the scale of 1 : 5.000.000 a plumbline deflection of θ seconds gives a distance of the isohypses of $206265/5000\ \theta = 41.3/\theta$ mm. In this way our map not only gives a picture of the geoid but it can likewise be used for reading the deflection of the plumbline over this whole area; it thus serves as a means of interpolating between the stations where θ has been computed.

According to the anomalies used, the geoid as well as the plumbline deflections refer to an Earth of which the topographic masses as well as the isostatic compensation according to the Hayford supposition have been taken away. This has been done in order to obtain a more regular result for both; we may expect that the results would otherwise have been found to be so locally variable that a much more detailed study and, therefore, a considerably more laborious system of computations would have been required.

TABLE I.

TABLE OF PLUMBLINE DEFLECTIONS FOR EAST INDONESIA.

First column = current number (missing numbers have been discarded for various reasons).

second column : S = Sea, L = land station (harbours counted as land).

third column : name.

fourth column : number of gravity station

fifth column : number of gravity profile

sixth and seventh column : geographical coördinates

eighth and ninth column : plumbline deflection to the East and North

tenth column : total plumbline deflection

eleventh column : angle of θ'' with the east direction (ENWS)

C = Control measurement (see page 13) where θ'' is mean of both measures.

P		Name	Stat.	Pr.	λ	β	θ_x	θ_y	θ	φ	
1	S		269	2	126°26′	+4°23′	−37.″9	+10.″2	39.″1	160°	C Weak
2	L	Talaud Is.	—	2	126°44′	+4°28′	−32.″4	+ 3.″6	32.″6	174°	
3	S		268	2	127° 3′	+4°33′	−14.″4	+ 3.″8	14.″9	165°	
4	S		—	2	127°30′	+4°40′	+12.″8	+ 7.″5	14.″1	30°.5	C
5	L	Sangi	—	A	125°37′	+3°44′	−20.″1	+12.″1	22.″8	149°	C Weak
6	L	Kaboeroeang	—	A	126°48′	+3°47′	−28.″7	+13.″0	31.″5	156°	
7	S		—	A	128°45′	+3°00′	+21.″4	+ 5.″2	22.″0	14°	
8	S	Siao	271	3	125°27′	+2°45′	−22.″3	+12.″9	25.″8	149°	C
9	S		—	3	125°50′	+2°35′	−35.″6	+16.″4	39.″4	155°	
10	S		272	3	126°14′	+2°26′	−28.″2	+ 8.″9	29.″6	162°.5	
11	S		—	3	126°50′	+2°12′	− 0.″4	− 1.″6	1.″6	(256°)	
12	S		342	A	120°46′	+1°23′	0.″0	+ 4.″0	4.″0	90°	
12a	S		—	A	123°45′	+1°35′	− 5.″8	+ 9.″4	11.″0	122°	
13	S		—	4	124°14′	+1°00′	− 6.″9	+ 8.″7	10.″2	128°.5	C
14	L	Menado	337	A	124°50′	+1°30′	−15.″1	+ 8.″5	17.″5	151°	C
15	L	Majo	—	H	126°21′	+1°19′	− 0.″5	− 3.″3	3.″3	(261°)	Weak
16	S		—	3	127°25′	+1°58′	+18.″9	−18.″8	25.″6	315°	C
17	L	Tobelo	262	3	128° 1′	+1°44′	+15.″8	−10.″6	19.″0	332° 5	C
18	L	Gorontalo	331	A	123° 3′	+0°30′	− 2.″2	+12.″1	12.″3	100°	
19	S		—	4	124°26′	+0°43′	− 9.″8	+12.″0	15.″1	129°	C
22	L	Ternate	—	H	127°23′	+0°47′	+12.″7	− 2.″6	13.″0	348°	
23	L	Weda	—	H	127°53′	+0°20′	+ 8.″7	− 0.″2	8.″7	359°	
24	L	Boeli	—	H	128°17′	+0°53′	+ 6.″6	− 2.″4	7.″0	340°	
25	L	Ngollopoppo	—	H	128°59′	+0°10′	+ 6.″5	+ 1.″4	6.″6	12°	
26	L	Ajoe Is.	—	H	131°11′	+0°34′	+ 2.″0	+ 4.″7	5.″1	67°	
27	L	Donggala	347	A	119°45′	−0°40′	− 3.″0	+ 3.″7	4.″8	129°	
29	L	Batoe Daka	—	A	121°45′	−0°27′	− 5.″9	+ 4.″4	7.″4	143°	
31	S		332	4	124°00′	−0°07′	−12.″4	+12.″8	19.″2	134°	C
32	S		—	4	124°18′	−0°24′	−16.″9	+ 3.″3	17.″2	169°	
33	S		324	4	124°36′	−0°40′	−12.″5	−10.″0	17.″3	219°	C
35	L	Laboeha	—	5H	127°29′	−0°38′	+12.″0	+ 1.″7	12.″1	8°	Weak
36	L	Siman	—	5H	128° 5′	−0°28′	+ 6.″4	+ 0.″6	6.″5	5°	

	P	Name	Stat.	Pr.	λ	β	θx	θy	θ	φ	
37	L	Garnei	—	5H	128°13′	—0°46′	+ 5.″5	+ 2.″3	6.″0	23°	
38	S		—	5	128°46′	—0°29′	+ 3.″5	+ 1.″7	3.″9	26°	
39	L	Gag	—	H	129°55′	—0°27′	+ 3.″8	+ 3.″4	5.″1	42°	
40	L	Waigeo	—	A	131° 0′	—0°20′	+ 0.″7	+ 6.″2	6.″2	84°	
41	L	Samatee	—	H	131° 4′	—0°58′	+ 1.″9	+ 6.″8	7.″0	74°	
42	L	Peling	—	A	123°10′	—1°13′	— 2.″8	— 6.″8	7.″4	248°	
43	L	Masoni	—	H	124° 8′	—1°45′	+ 5.″7	—15.″8	16.″8	290°	
44	S		—	4	124°50′	—1° 6′	— 6.″4	—36.″5	37.″0	260°	C_1
45	S		—	4	125° 5′	—1°32′	— 3.″8	—36.″8	37.″0	264°	
46	L	Soela	279	4	125°19′	—1°57′	— 2.″5	—19.″9	20.″1	263°	
47	S		—	5	125°41′	—1°38′	— 9.″8	—39.″1	40.″3	256°	C_1
48	S		278	5	126° 3′	—1°20′	— 6.″9	—38.″9	39.″5	260°	C_1
49	S		—	5	126°28′	—1°10′	+ 4.″8	—23.″4	27.″5	280°	C_1
50	L	Lifoematoela	—	H	126°59′	—1°48′	— 0.″4	—19.″0	19.″0	269°	Weak
51	L	Beling	—	H	127°26′	—1°19′	+ 6.″5	— 0.″9	6.″6	352°	
52	L	Goemoemoe	—	H	127°38′	—1°50′	— 1.″8	— 4.″5	4.″8	248°	Weak
53	L	Waigama	—	H	129°49′	—1°50′	+ 1.″7	+ 6.″3	6.″5	75°	
54	S		—	6	131°10′	—1°22′	+ 2.″3	+ 4.″9	5.″4	65°	Strong
55	L	Cape Williams	353	A	118°45′	—2°35′	— 2.″6	+ 2.″1	3.″3	141°	
56	S		—	A	123° 0′	—2°20′	+ 3.″9	— 6.″9	7.″9	300°	
57	S		280	4	125° 8′	—2°52′	+ 0.″4	+ 1.″0	1.″1	(68°)	
60	S		—	A	128°15′	—2° 0′	+ 0.″9	+ 2.″0	2.″2	(66°)	
61	L	Wahai	—	H	129°29′	—2°48′	— 2.″1	+ 1.″6	2.″6	143°	Weak
62	S		—	A	130° 0′	—2°15′	+ 1.″3	+ 9.″0	9.″1	82°	
63	S		244	6	130°54′	—2°38′	+ 0.″9	+12.″6	12.″6	86°	
64	S		245	6	131° 0′	—2°17′	+ 1.″0	+ 9.″0	9.″0	84°	
64a	L	Fak-Fak	—	A	132°16′	—2°56′	+ 2.″4	+ 5.″2	5.″7	65°	
64b	L	Aroba	—	A	133°23′	—2°33′	— 5.″0	— 2.″9	5.″8	210°	*
64c	L	Steenkool	—	A	133°32′	—2° 7′	— 6.″8	+ 1.″6	6.″9	167°	
65	S		—	A	120°40′	—3°35′	— 1.″2	— 4.″7	4.″8	256°	
66	S		—	A	122°50′	—3°20′	+ 8.″3	— 3.″5	8.″7	335°	C
67	S		—	4	125°20′	—3°30′	— 2.″4	+ 3.″0	3.″8	129°	
69	L	Wapotik	—	H	126°36′	—3° 4′	—11.″4	+ 4.″1	11.″3	160°	C
70	L	Namrole	—	H	126°44′	—3°52′	— 8.″1	— 3.″3	8.″8	202°	
71	L	Saroma	—	H	127°13′	—3°40′	— 6.″9	— 4.″2	7.″7	211°	C
72	L	Ambon	—	H	128°11′	—3°41′	— 4.″0	— 9.″5	10.″0	247°	C
74	L	Boela	—	H	130°30′	—3° 6′	— 3.″6	+ 2.″4	4.″3	146°	
75	L		243	6	130°52′	—3°29′	— 6.″3	— 7.″4	8.″6	230°	C
77	L	Geser	—	6H	130°54′	—3°52′	—14.″8	—12.″4	19.″3	220°	C
77a	L	Argoeni Bay	—		132°29′	—3° 7′	— 1.″0	— 3.″7	3.″8	254°	*
78	L		233	7	133°40′	—3°45′	— 1.″1	+ 1.″8	2.″1	122°	
79	L	Pare Pare	356	A	119°37′	—4° 0′	— 0.″6	— 1.″2	1.″4	(243°)	
80	L		306	A	120°28′	—4°45′	+ 2.″6	— 2.″7	3.″8	314°	
81	L	Kolaka	309	A	121°35′	—4° 3′	+ 0.″3	— 5.″4	5.″4	273°	
82	S		318	11/12	124°12′	—4°32′	+ 3.″4	+ 4.″7	5.″8	54°	

*) Mainly from New Guinea gravity observations.

P		Name	Stat.	Pr.	λ	β	θx	θy	θ	φ	
84	S		284	4	126° 1′	—4°32′	— 3.″7	— 0.″6	3.″8	189°	
84a	S		—		127°50′	—4°40′	+ 1.″8	— 4.″0	4.″4	294°	
85	L	Banda	157	6H	129°54′	—4°31′	— 4.″1	+ 1.″0	4.″2	166°	
86	S		—	6	130° 8′	—4°22′	— 5.″0	— 1.″7	5.″3	199°	
87	S		242	6	130°22′	—4°13′	— 8.″9	— 5.″2	10.″3	210°	C
89	S		—	7	131° 2′	—4°53′	—14.″9	+ 2.″4	15.″1	171°	
90	S		238	7	131°42′	—4°45′	—11.″6	+ 8.″3	13.″7	144°	C
91	S		—	7	132° 3′	—4°38′	+ 1.″9	+ 7.″1	7.″4	75°	
92	S		237	7	132°24′	—4°31′	+11.″0	+10.″6	15.″3	44°	C_1
93	S		—	7	132°50′	—4°19′	+ 8.″1	+ 8.″8	12.″0	54°	
94	L		236	7	133°15′	—4° 7′	— 1.″4	+ 3.″0	3.″3	115°	
95	L	Makassar	357	A	119°24′	—5° 8′	+ 0.″9	+ 1.″1	1.″4	(51°)	
96	L		305	A	120°27′	—5°44′	+ 4.″6	+ 2.″8	5.″4	31°	
97	S		—	A	122°10′	—5°40′	— 4.″0	— 1.″4	4.″2	199°	
99	L	Boeton	312	A	122°37′	—5°27′	— 0.″2	— 0.″1	0.″2	—	
101	S		—	12	124°12′	—5°15′	+ 2.″3	+10.″1	10.″4	77°	
102	S		317	11	125° 6′	—5° 7′	— 1.″7	+ 7.″5	7.″7	103°	
103	S		—	11	125° 8′	—5°50′	+ 2.″2	+ 4.″5	5.″0	64°	
104	S		—	10	127°40′	—5°42′	— 0.″8	— 0.″9	1.″2	(228°)	
104a	S		—	A	128°25′	—5°40′	— 2.″4	— 1.″7	3.″0	214°	Strong
105	S		158	7*	129°28′	—5°36′	— 6.″0	+ 0.″9	6.″1	171°	
106	S		—	6	129°41′	—5° 4′	— 6.″8	— 2.″2	7.″1	198°	
108	L	Manoek	—	7/8H	130°18′	—5°33′	—10.″9	+ 3.″2	11.″4	163°	
111	S		227	8	131° 8′	—5°36′	—26.″9	+ 9.″9	29.″2	160°	C
112	S		—	8	131°42′	—5°38′	— 3.″0	+ 5.″9	6.″6	117°	
113	S		228	8	132° 5′	—5°39′	+10.″9	+ 3.″4	10.″7	17°	C
114	L	Toeal	229	8	132°43′	—5°38′	+ 6.″5	+ 3.″4	6.″8	28°	C
116	S		313	12	122°58′	—6°25′	— 0.″3	— 1.″2	1.″2	(156°)	
117	S		—	12	124°12′	—5°58′	+ 1.″5	+ 4.″8	5.″0	73°	
118	S		—	11	125°37′	—6°36′	+ 2.″1	+ 0.″1	2.″1	(3°)	
119	S		—	11	125°55′	—6°51′	+ 0.″7	+ 5.″0	5.″0	82°	
119a	S		—	A	126°50′	—6°10′	+ 1.″0	+ 1.″0	1.″4	(45°)	Strong
120	L	Nila	—	H	129°31′	—6°45′	— 7.″9	+10.″6	13.″7	127°	C
121	L	Seroea	224	9H	130° 1′	—6°18′	—10.″8	+ 7.″9	14.″0	144°	C
121a	L		—	A	120°40′	—7°10′	+ 2.″2	+ 3.″4	4.″0	57°	
122	S		292	12	123°24′	—7°28′	— 0.″3	+ 7.″4	7.″4	92°	
123	S		291	A	124°40′	—7°20′	— 0.″4	+ 7.″9	7.″9	93°	
124	L	Wetar	—	11	125°50′	—7°51′	— 3.″2	+24.″1	24.″3	97.5°	
125	L	Djoeka	—	H	127°38′	—7°36′	— 3.″4	+21.″3	21.″6	99°	
127	L	Damar	—	10H	128°41′	—7° 8′	— 5.″5	+16.″4	17.″4	108.5°	C
129	L	Wetan	—	H	129°31′	—7°51′	— 7.″4	+17.″6	18.″6	113°	C
130	L	Dai	—	H	129°43′	—7°34′	— 7.″2	+18.″7	20.″0	111°	
131	S		223	9	130°24′	—7° 5′	—15.″8	+12.″2	20.″0	142°	
132	S		—	9	130°35′	—7°22′	—14.″8	+13.″6	20.″1	137.5°	
133	S		222	9	130°46′	—7°39′	— 6.″1	+ 4.″2	7.″4	145°	

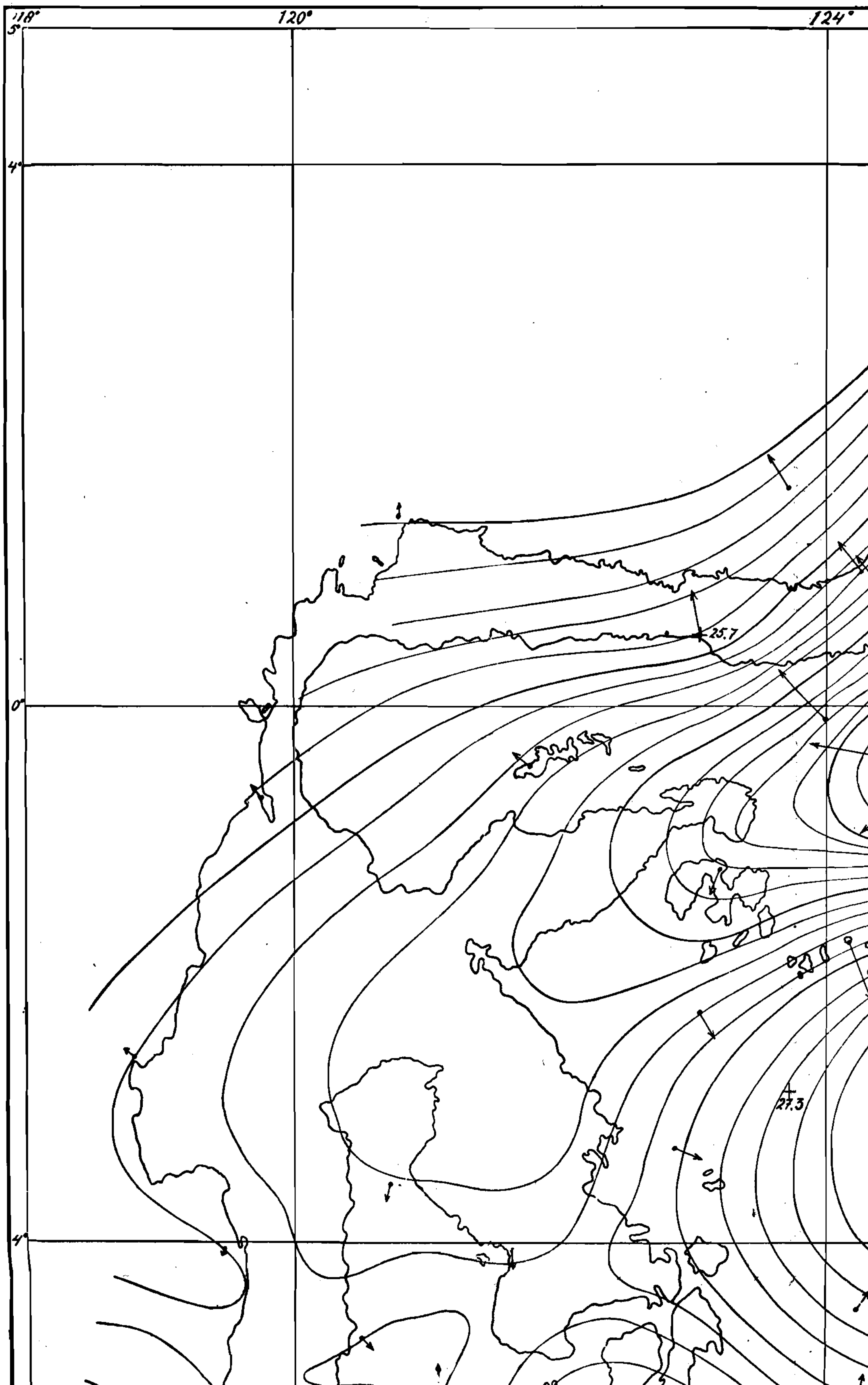
*) Profile 6, 7, 8 and 9.

P		Name	Stat.		λ	β	θ_x	θ_y	θ	φ
136	L	Sangeang	—	A	119° 5′	—8°15′	+ 1.″3	— 1.″4	1.″9	(313°)
137	S		—	A	120° 0′	—8° 0′	+ 0.″9	+ 1.″5	1.″8	(59°)
138	L		—	A	121°35′	—8°40′	— 4.″8	+ 8.″2	9.″5	120°
139	S		—	A	121°45′	—8° 0′	+ 0.″6	+ 9.″2	9.″2	86°
140	L		—	12	122°40′	—8°37′	— 5.″1	+16.″7	18.″1	107°
141	L	Dilly	—	A	125°34′	—8°35′	— 4.″0	+19.″2	19.″6	102°
142	L		—	11	126°21′	—8°30′	— 2.″7	+18.″8	19.″0	98°
143	L		—	11	126°37′	—8°50′	+ 0.″1	+ 6.″5	6.″5	90°
144	L	Sermata	—	10H	128°50′	—8°12′	— 2.″5	+15.″3	15.″5	99°
145	L	Sasar	—	A	119°58′	—9°18′	— 0.″4	+ 2.″5	2.″5	(99°)
146	S		210	12	122°30′	—9°35′	— 7.″6	+ 8.″8	11.″7	131°
149	S		216	11	126°37′	—9°13′	+ 6.″4	— 4.″0	7.″5	328°
152	L	Koepang	211	12	123°32′	—10°8′	— 3.″2	— 1.″2	3.″4	201°
153	S		212	12	123°48′	—10°27′	— 1.″2	— 7.″3	7.″4	261°

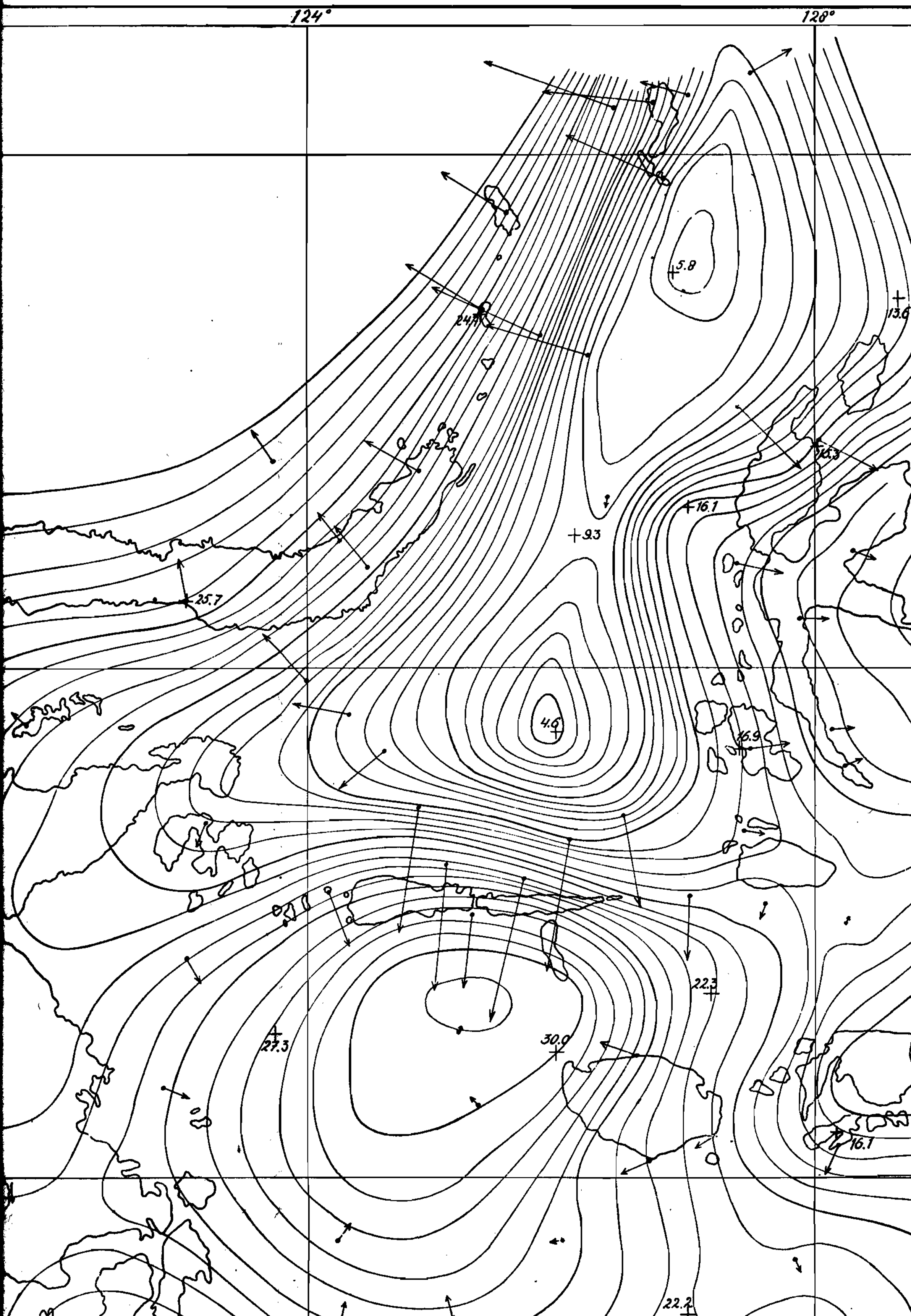
TABLE II.

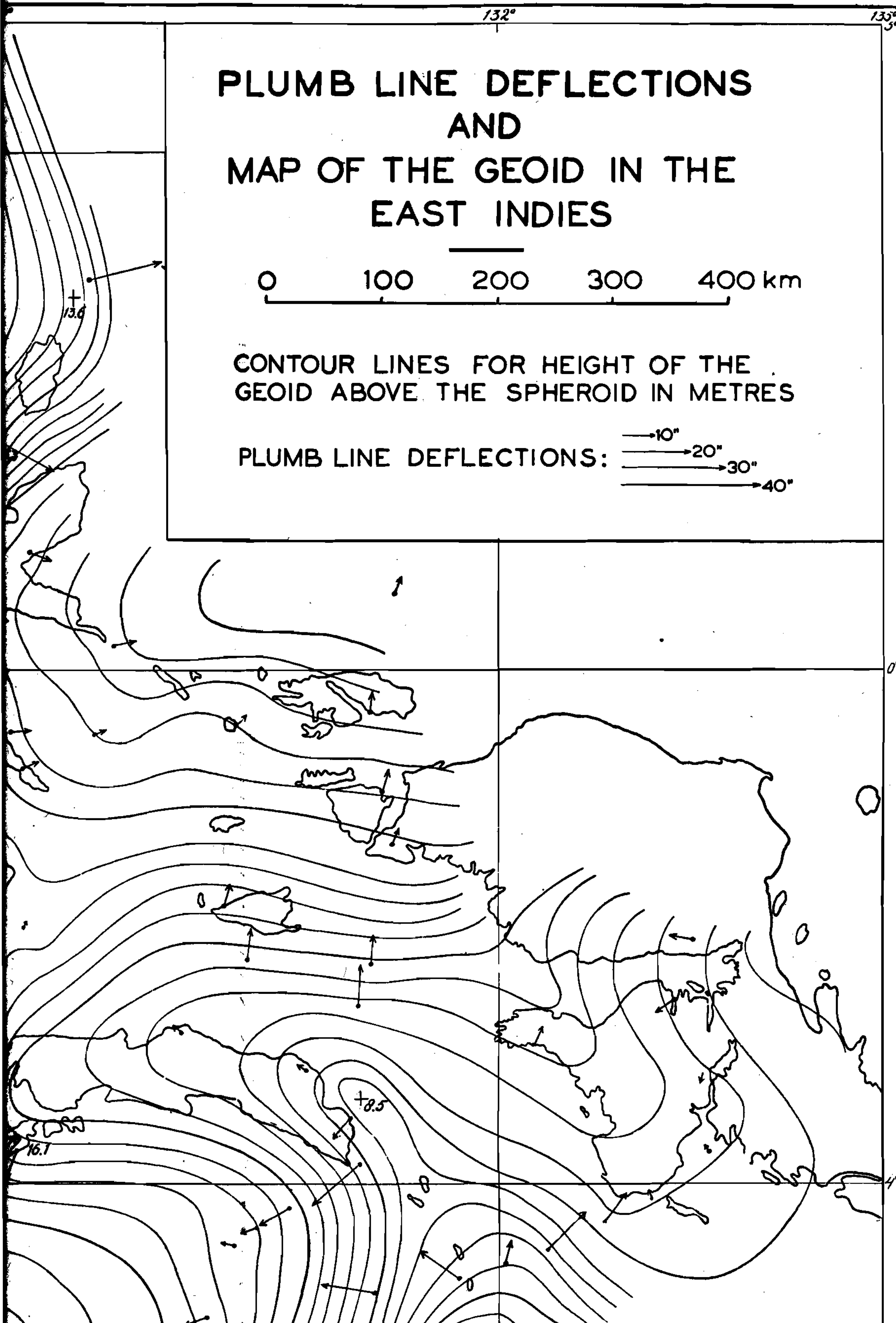
Stations for which N has been computed.

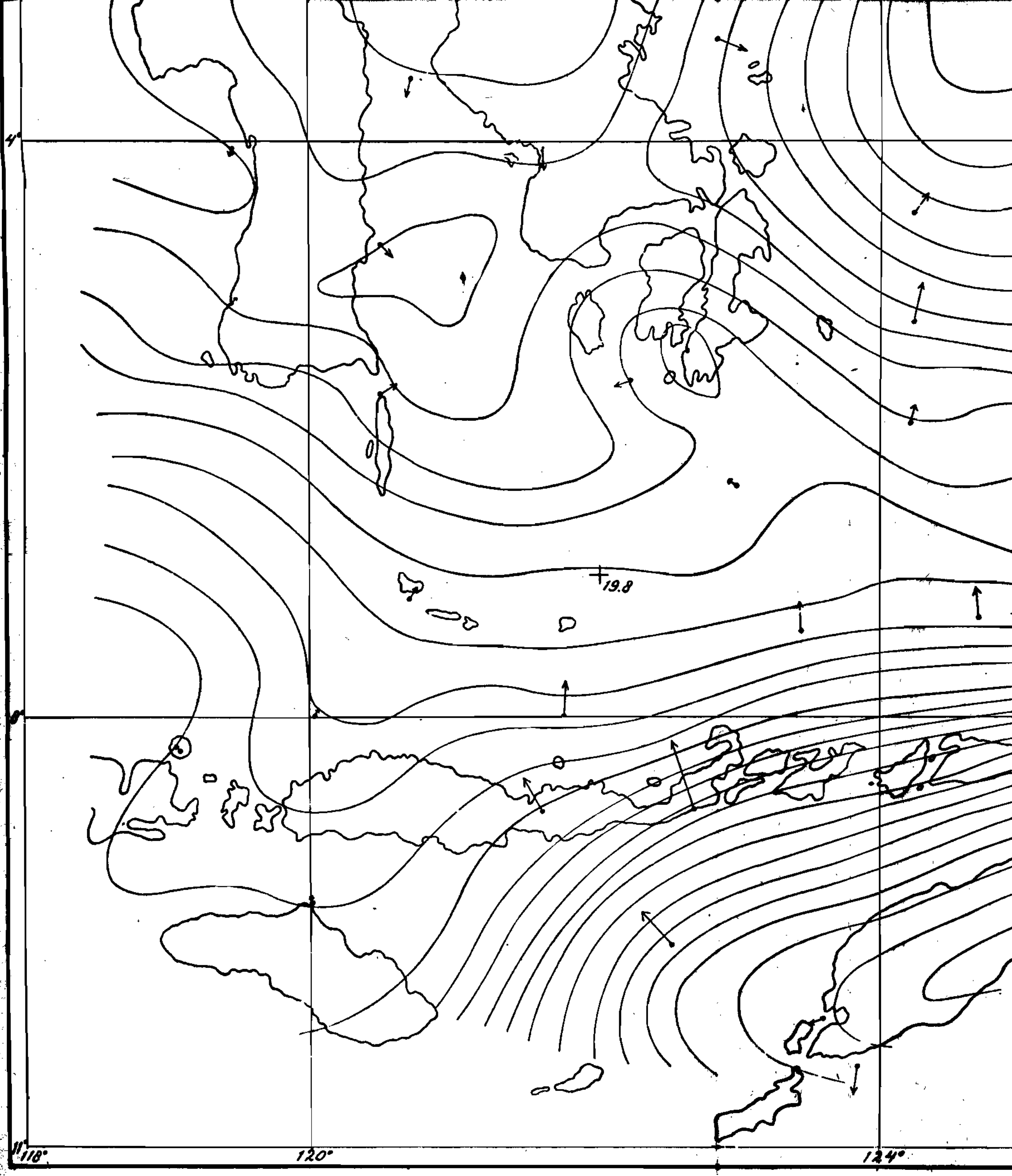
No	Latitude	Longitude	N in meters
1	0° 29′ S	125° 59′ E	+ 4.6
2	3 0 S	126 0 E	+30.0
3	7 0 S	122 0 E	+19.8
4	6 50 S	127 14 E	+21.4
5	5 39 S	132 5 E	— 1.8
6	1 45 S	126 57 E	+17.6
7	2 35 S	127 12 E	+22.3
8	3 41.3 S	128 10.4 E	+16.1
9	3 29.5 S	130 51.5 E	+ 8.5
10	1 43.7 N	128 0.7 E	+15.3
11	2 49 N	128 34 E	+13.6
12	2 44.6 N	125 27.5 E	+24.7
13	1 12 N	127 0 E	+16.1
14	0 38.5 S	127 27.2 E	+16.9
15	5 6 S	127 0 E	+22.2
16	2 51 S	123 43 E	+27.3
17	5 58 S	124 12 E	+21.9
18	0 29.7 N	123 3.4 E	+25.7
19	1 29.8 N	124 50.1 E	+30.7
20	1 0 N	126 9 E	+ 9.3
21	3 0 N	126 53 E	+ 5.8



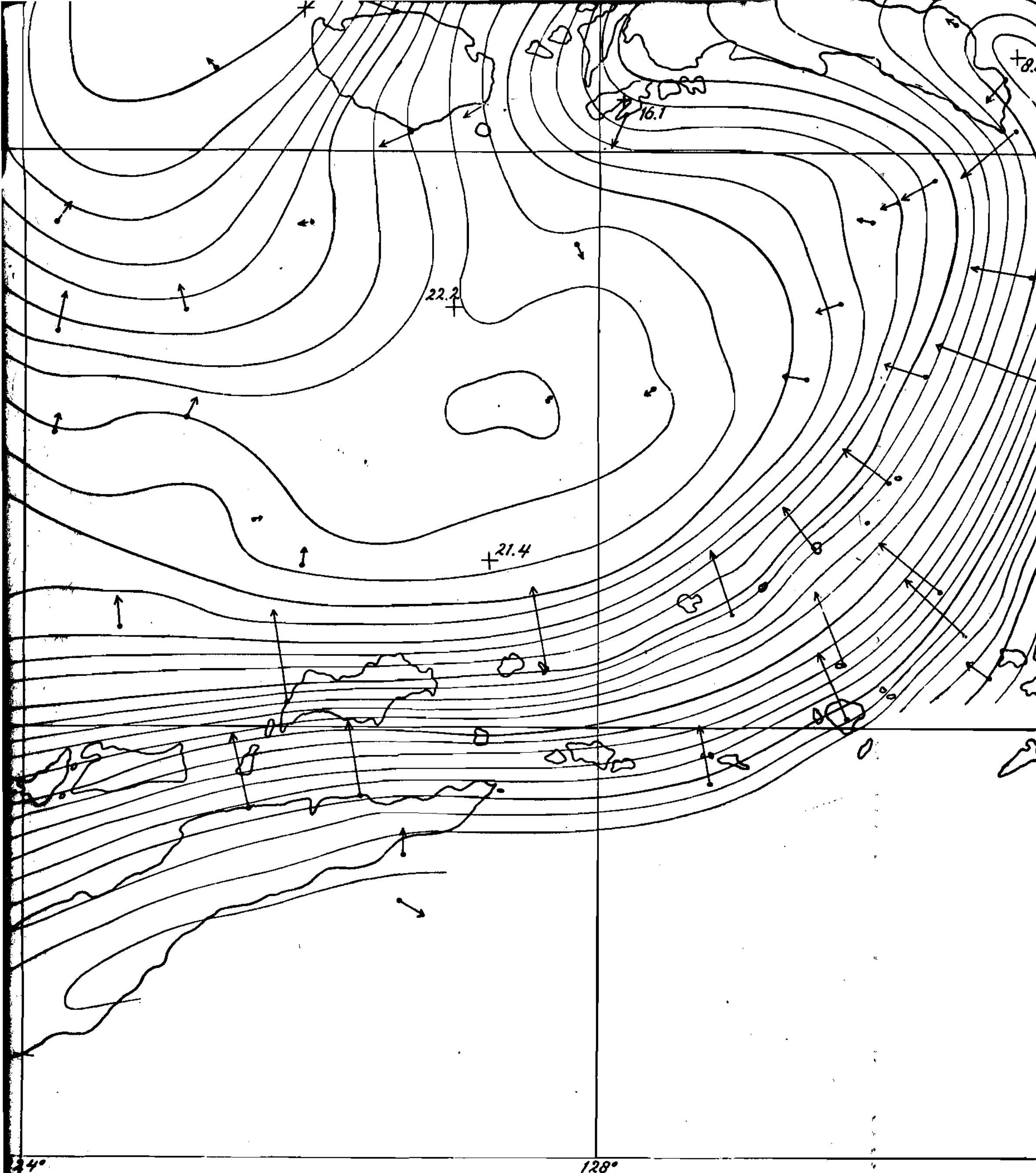
118°
120°
124°
5°
4°
0°
4°
25.7
27.3


124°
128°
5.8
13.6
24.7
16.1
9.3
25.7
16.3
4.6
16.9
22.3
27.3
30.9
16.1
22.2

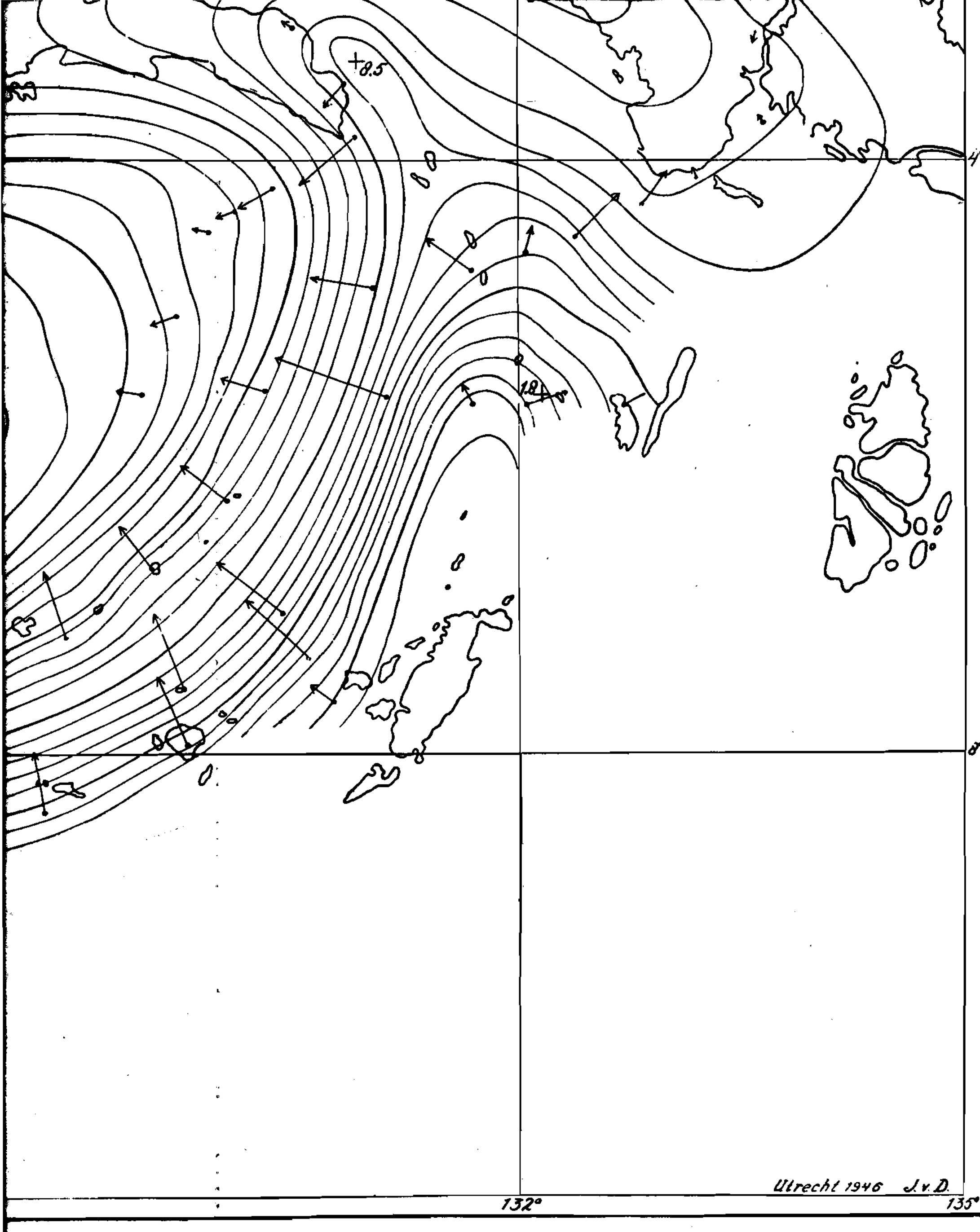

PLUMB LINE DEFLECTIONS
AND
MAP OF THE GEOID IN THE
EAST INDIES
0 100 200 300 400 km
CONTOUR LINES FOR HEIGHT OF THE
GEOID ABOVE THE SPHEROID IN METRES
PLUMB LINE DEFLECTIONS:
10"
20"
30"
40"
132°
135°
5°
0°
4°
13.6
8.5
16.1



4°
19.8
8°
118°
120°
124°



16.1
22.2
21.4
24°
128°



+8.5
1.8
4°
8°
Utrecht 1946 J.v.D.
132°
135°