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THE TWIN LAPLACE POINT UBACHSBERG-TONGEREN, APPLYING THE BLACK METHOD

by

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LIST OF THE SYMBOLS AND CONSTANTS

symbol	*meaning*
φ, φ_g	latitude (astronomic or geodetic)
λ, λ_g	longitude (positive west of Greenwich)
A, A_g	azimuth of the reference mark
T_k	times observed by the eye-piece micrometer ($k = 1, 2, \ldots N$)
ΔT	clock correction to *UTC* including travel time of radio signal and delay in the equipment used in the field
β	width of the contact strips
τ	lost motion of the micrometer screw
M	reading of the suspension level: $M = \frac{1}{2}(l+r)$
M_0	centre point of the level. Mean value approximately: $\frac{1}{2}[M_L+M_R]$
p	sensitivity of the suspension level
S_0'	apparent sidereal index
n'	change in the equation of equinoxes
φ_s'	circle reading at the star's transit
φ_t'	circle reading to the reference mark
t	local hour angle
LST	Local Sidereal Time
z	zenith distance of the star
a	azimuth of the star
q	parallactic angle of the star
l	quasi-observation
s	number of stars ($i = 1, 2, \ldots, s$)
N	number of contacts ($k = 1, 2, \ldots, N$)
α	apparent right ascension
δ	apparent declination

constants

$p = 1''.03$ per 2 mm (suspension level nr. 434)

$[da_k^2]/2N\varrho = 0''.043$ for $N = 27$ contacts

Angle value of the micrometer screw: $1''.532$ per division

THE TWIN LAPLACE POINT UBACHSBERG-TONGEREN, APPLYING THE BLACK METHOD

Summary

In the summer of 1968 astronomical observations were carried out at the primary stations Ubachsberg (The Netherlands) and Tongeren (Belgium) in order to make a twin Laplace point of these two stations. This work was undertaken in behalf of the New Adjustment of the European Triangulation. The latitude, longitude, and azimuth were determined simultaneously applying Blacks method [1] and using a Wild T4 universal theodolite. The results obtained are shown in section 8.

The main purpose of the measurements was to determine the geodetic azimuth $A_g = A+(\lambda-\lambda_g)\sin\varphi_0$, which was obtained with high accuracy, because of the strong negative correlation between the astronomical azimuth and astronomical longitude.

The misclosure between the two stations of this twin Laplace point amounts to 0''.57.

Furthermore, the accuracy requirements for Laplace points in The Netherlands were investigated. Based on the deviation of the geodetic part of the Laplace equation, it was found that a standard deviation of 0''.25 in the geodetic azimuth suffices.

The observer's personal equation was determined using an artificial moving star.

1 Introduction

A relation between geodetic and astronomical quantities is given by the well-known Laplace equation *):

$$A_g = A+(\lambda-\lambda_g)\sin\varphi_0 \quad . \quad . \quad .

Some disadvantages of the Black method, such as the rather complicated star selection and the influence of systematic errors in circle divisions of the instruments used, can be overcome without any difficulty. Star selection can be done most conveniently using a computer, and the generally small systematic errors in circle divisions of modern instruments can be accurately determined applying the new methods developed in this field.

This paper gives a description of how the Black method was applied in The Netherlands and discusses the results obtained at the twin Laplace point Ubachsberg-Tongeren. The assistance of the "Bijhoudingsdienst van de Rijksdriehoeksmeting" (Netherlands Triangulation Service) in this project is gratefully acknowledged. The organization of the fieldwork and the preparations at the stations were largely the responsibility of Mr. M. HAARSMA, while Mr. D. L. F. VAN LOON carried out the observations with great care and ability.

2 The Black method

With the Black method a number of stars are observed in vertical transit. For theoretical and practical reasons the stars are observed at approximately equal zenith distances and regularly distributed in azimuth.

Applying the cotangent rule in the position triangle gives:

$$\cot a_i = \cot(A+\psi_i) = \frac{\sin\varphi\cos(t_i^G-\lambda) - \cos\varphi\tan\delta_i}{\sin(t_i^G-\lambda)} \quad \ldots\ldots\ldots \quad (2)$$

where: A = unknown azimuth of the reference mark
ψ_i = the measured horizontal angle between the reference mark and the direction to the stars: $\psi = \varphi_s - \varphi_t$
t_i^G = the Greenwich hour angle of the stars

After introducing φ_0, λ_0, and A_0 as approximate values for the unknowns and differentiating (2), we obtain the correction equations:

$$\Delta\varphi \sin a_i' \cot z_i' + \Delta\lambda \cos\varphi_0 \cos a_i' \cot z_i' - \Delta A_g = -(\bar{l}_i + \bar{v}_i) \quad (i = 1, 2, \ldots, s) \quad (3)$$

in which:

$$l_i = a_i' - (A_0 + (\varphi_s)_i - (\varphi_t)_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

is the so-called quasi-observation.

The star's azimuth is computed with the approximate values:

$$\cot a_i' = \frac{\sin\varphi_0\cos(t_i^G-\lambda_0) - \cos\varphi_0\tan\delta_i}{\sin(t_i^G-\lambda_0)} \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

while the zenith distance z_i' is computed from:

$$\cos z_i' = \sin\varphi_0\sin\delta_i + \cos\varphi_0\cos\delta_i\cos(t_i^G-\lambda_0) \quad \ldots\ldots\ldots\ldots \quad (6)$$

The Greenwich hour angle of the stars is computed in the following way. Using an eye-piece micrometer, the star is tracked with the vertical wire over a number of contacts that are symmetrical with respect to the fixed central wire. The moments T_k of making contact are registered by a chronograph and the mean value of these times is computed from (omitting the index i):

$$\overline{T} = \frac{[T_k]}{N} \qquad (k = 1,2,\ldots N) \quad . \quad (7)$$

To this mean value of the times some corrections are applied:

$$UT1 = \overline{T} + \Delta T + (UT1 - UTC) + \frac{\beta'' + \tau''}{2 \times 0.997 \times 15} |\sec \delta \sec q| \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

in which:

ΔT = clock correction to be applied to UTC

$UT1$–UTC = correction to $UT1$ as published in the Bulletin Horaire

β'' = width of the contact strips

τ'' = lost motion of the micrometer screw

q = parallactical angle of the star

The $UT1$ obtained in (8) is transformed into Greenwich apparent sidereal time:

$$GAST = UT1 + (UT1)^{s}0.0027379 + S_0' + n' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

in which:

S_0' = apparent sidereal index

n' = change in the equation of equinoxes

From (9) follows the Greenwich hour angle:

$$t^G = GAST - \alpha \quad . \quad (10)$$

The quantities α, δ in the above formulae are the interpolated star coordinates, corrected for the influence of the short periodic nutation. Some corrections are also applied to the other observation quantities. The circle reading φ_s' at the star's transit is corrected for the periodic circle error, for the inclination of the trunnion axis, for the daily aberration, and for the curvature of the star's path:

$$\varphi_s = \varphi_s' - R(\varphi_s') \pm p(M - M_0)\cot z - 0''.32 \cos\varphi \operatorname{cosec} z \cos a + (C_1' + C_2')\frac{[da_k^2]}{2N\varrho} \qquad (11)$$

where: $C_1' = (\cos z \tan q + \cot a \tan^2 q)\operatorname{cosec}^2 z$

$C_2' = -2 \tan q \cos z \operatorname{cosec}^2 z$

The upper sign in (11) refers to instrument position face left, the lower sign to face right.

The circle reading φ_t' at the reference mark is corrected for the circle error:

$$\varphi_t = \varphi_t' - R(\varphi_t') \quad . \quad (12)$$

Substitution of (5), (11) and (12), into (4), yields one quasi-observation for each series of measurement. Each star is measured in two or four series, with the instrument in different positions, in order to eliminate the systematic instrumental errors. The quantity $\bar{l}_i$ in (3), to be introduced into the adjustment, is the mean value of the number of series for each star:

$$\bar{l}_i = \frac{[l_{ij}]}{n} \qquad \begin{array}{l} j = 1,2 \text{ for FL} \\ j = 3,4 \text{ for FR} \\ n = \text{number of series} \end{array} \qquad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

which are usually introduced into the computation with equal weights.

From the correction equations (3):

$$Ax = -(\bar{l}+\bar{v}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

follow the normal equations:

$$A^*Ax = -A^*\bar{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

Solution of these normal equations gives the unknowns:

$$x = \begin{pmatrix} \Delta\varphi \\ \Delta\lambda \cos\varphi_0 \\ \Delta A_g \end{pmatrix} = -(A^*A)^{-1}A^*\bar{l} = -QA^*\bar{l} \quad \ldots\ldots\ldots\ldots \quad (16)$$

Finally,

$$\left.\begin{array}{lll} \varphi &= \varphi_0+\Delta\varphi & -(x\cos\lambda_0 + y\sin\lambda_0) \\ \lambda &= \lambda_0+\Delta\lambda & -(x\sin\lambda_0 - y\cos\lambda_0)\mathrm{tg}\varphi_0 \\ A_g &= A_0+\Delta A_g+(\lambda_0-\lambda_g)\sin\varphi_0 & +(x\sin\lambda_0 - y\cos\lambda_0)\cos\varphi_0 \\ A &= A_0+\Delta A_g-\Delta\lambda\sin\varphi_0 & +(x\sin\lambda_0 - y\cos\lambda_0)\sec\varphi_0 \end{array}\right\} \quad (17)$$

in which the corrections for polar motion are added to the quantities to be determined. The coordinates x and y of the true pole refer to the Conventional International Origin and are to be taken in units of seconds of arc. It should be noted that, in case an arbitrary value for the longitude is introduced, the correction $(\lambda_0-\lambda_g)\sin\varphi_0$ must be applied in order to obtain the geodetic azimuth. The only difficulty is that the geodetic longitude usually refers to the origin of the triangulation system (in The Netherlands: Amersfoort) and not, like the astronomical longitude, to Greenwich. This difficulty can be overcome by adopting a fictitious geodetic longitude consisting of two components: the geodetic longitude of the station in question counting from the origin of the triangulation system, plus the astronomical longitude of the origin counting from Greenwich.

The same results may be obtained by introducing fictitious geodetic latitude and longitude as approximate values into the adjustment according to BLACK [1], however, in case of a large deviation of the vertical, closer approximate values should be preferred.

For the determination of the standard deviation of the unknowns, the corrections are computed by substituting the unknowns into (14):

$$\bar{v} = Ax-\bar{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

The estimation of the variance factor is:

$$\bar{\sigma}_0^2 = \frac{[\bar{v}_i^2]}{s-3} \quad \ldots \quad (20)$$

and the standard deviations are:

$$\sigma_x = \bar{\sigma}_0 \sqrt{Q_{xx}} \quad \ldots \quad (21)$$

in which Q_{xx} is approximately a diagonal matrix according to (16). However, deriving the standard deviation from the corrections to the *single* quasi-observations is to be preferred. Assuming that each star is observed in four series, we obtain:

$$v_{ij} = \bar{l}_i + \bar{v}_i \pm K - l_{ij} \qquad \begin{matrix} \text{FL:} & + \\ \text{FR:} & - \end{matrix} \quad \ldots \quad (22)$$

in which K is a constant instrument error, consisting of the collimation error, the influence of inclination of the horizontal axis, and the deviation between the zero position of the moving wire and the fixed central wire. The quantity K is considered as a fourth unknown, which can be computed independently, assuming four series are measured:

$$K = \frac{1}{4s}[l_{i1} + l_{i2} - l_{i3} - l_{i4}] \quad \ldots \quad (23)$$

The estimation of the variance with respect to a single observation is then:

$$\sigma_0^2 = \frac{[v_{ij}^2]}{4s-4} \quad \ldots \quad (24)$$

from which follow the standard deviations:

$$\sigma_x = \sigma_0 \sqrt{\tfrac{1}{4} Q_{xx}} \quad \ldots \quad (25)$$

3 Selection of stars

In order to determine the three unknowns free of correlation, the stars should be distributed regularly in azimuth (see appendixes 2 and 3). A Telefunken TR4 computer was used for the preparation of the star list preliminary to the selection of stars. All the suitable stars of the Apparent Places of Fundamental Stars ($\delta > -20°$) were punched on tape with the following data: number of the star, magnitude, right ascension, declination. Two other data were added: the latitude of the station in question and the zenith distance ($z = 60°$), at which the stars should be observed. With these data as input, the computer produced a star list containing the data necessary for the selection of the stars: number of the star, magnitude, *LST*, azimuth, parallactic angle (q), velocity (v), horizontal velocity (v_H), $\mathrm{d}a/\mathrm{d}t$, $\mathrm{d}z/\mathrm{d}t$

(printed in sequence of *LST*). The stars are not generally suited for observing with an eyepiece micrometer at any position. If the horizontal velocity component of a star is:

$$v_H = 15 \cos\delta \cos q < 5''/\text{sec} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (26)$$

it will be rather difficult for the observer to track the star with the moving wire. With $\varphi = 51°$ and $z = 60°$ this problem does not arise, because the lowest horizontal velocity shown in the star list is 5.5″/sec. Moreover, stars with magnitude < 2.5 and > 5.5 were not included in the measuring programme.

In addition to the star list, the computer produced a *diagram*, in which the azimuth (the four quadrants overlapping each other) were plotted on the horizontal axis and the *LST* on the vertical axis. It contained all the stars of the star list with an indication of their quadrants: 1, 2, 3, or 4. By means of this diagram four matching stars can be selected as follows:

1st quadrant: a
2nd quadrant: $a + 90°$
3rd quadrant: $a + 180°$
4th quadrant: $a + 270°$

In selecting four stars the following tolerances were allowed:

- in *LST* a minimum time interval of 25^m between the consecutive stars, in view of the duration of the measuring programme;
- in azimuth a maximum deviation of $\pm 5°$.

One measuring programme can contain more sets of four stars, all independent of each other. In order to find the selected stars on the celestial sphere the necessary data (*LST*, a, etc.) were taken from the star list. An accuracy of 1^m for the *LST*, 1′ for the azimuth, and 1′ for the zenith distance is sufficient for this selection. The azimuthal and zenithal velocities (da/dt and dz/dt) of the star list can be used for finding the star again after changing the face of the telescope.

4 Instruments

The measurements were carried out using a universal theodolite Wild T4 (1957). It should be emphasized that the instrument must be adjusted very carefully in all respects according to the instructions given in [2].

Special attention should be paid to the mechanical stabilization of the trunnion or horizontal axis. The trunnion axis rests, by means of cylindrical pivots, on four carrying bearings. To protect the pivots against wear caused by the heavy weight of the telescope (18 kg), it is also provided with two spring bearings. The latter relieve the weight on the four carrying bearings. Laboratory tests have shown that the residual weight of the telescope on the carrying bearings must be adjusted very carefully. Favourable results may be obtained with a total residual weight of about 4 to 5 kg which can be roughly established by direct measurement. This must be followed by a fine adjustment for balancing the horizontal axis precisely. Of importance is the fact that, in case of an unbalanced horizontal axis, the line of sight is also displaced in the horizontal plane when the vertical slow motion screw is turned

left or right. Consequently, the fine adjustment can be carried out by eliminating this systematic displacement empirically. It seems rather strange that the instruction books do not mention this important adjustment.

Another remark concerns the eye-piece micrometer, which had to be renewed owing to the poor quality of the contact strips and a varying lost motion. On special request the contact drum of the new eye-piece micrometer can be provided with only 9 contact strips, with one double interval for the identification of the zero point.

Several components of the T4 were tested before the observations were carried out. These tests referred to:

- roundness of the horizontal axis;
- systematic errors of the horizontal circle division;
- sensivity of the suspension level;
- width of the contact strips and lost motion of the eye-piece micrometer.

The roundness of the horizontal axis was investigated by J. C. DE MUNCK, who used the method he had himself developed [3, 4, 5]. Applying this method only slight deviations of the circular shape were found, resulting in a maximum deviation of 0''.1 in horizontal direction. Deviations of this magnitude can be neglected.

The horizontal circle division was investigated according to Roelofs's method – bundle of rays method – described in [6]. The systematic error in the circle division was determined from two independent measurements, taking into account 14 terms of the Fourier series. The results of this investigation were published in [7]. From the residual errors the following standard deviations were obtained:

	σ_s	$\sigma_{R(\varphi)}$
measurement of 23rd – 24th November 1966	0''.17	0''.06
measurement of 12th – 13th December 1966	0''.22	0''.06

The average systematic circle errors $R(\varphi)$ are shown in Appendix 1.

The suspension level was investigated by means of a level trier. The results indicate an average sensitivity of 1''.03 per division (standard deviation 0''.07) with no indication of systematic errors.

The width of the contact strips and the lost motion were determined from three complete measurements, applying the method described in [2]. The resulting average value, multiplied by the micrometer's angle value, is:

$$\beta'' + \tau'' = 1.30 \times 1''.532 = 1''.992$$

The Department of Electrical Engineering of the Delft University of Technology built a special transistorized apparatus, which was named *Chronofix*, for automatic recording of radio time signals on the Omega chronograph.

In addition to the radio receiver, a quartz clock, HBG receiver, and mechanical contacts can be connected to the Chronofix. The eye-piece micrometer was connected to the mechanical contacts. In order that the Omega Chronograph might be used, the making contacts were changed into breaking contacts.

5 Testing of the equipment

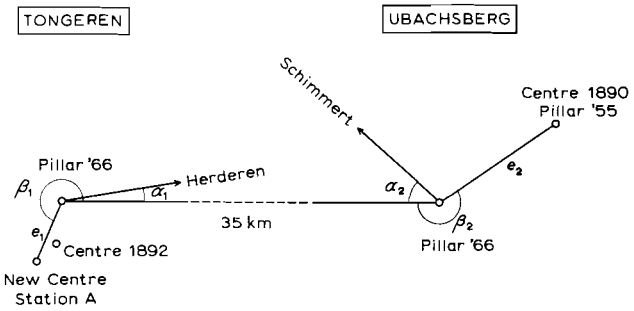
To investigate the reliability of the instruments and to give the observer an opportunity to become familiar with the measuring programme, test measurements were carried out in Delft in 1968 before the actual observations in Ubachsberg and Tongeren were made. The T4 was mounted on a pillar and two programmes of 8 stars were observed. The results of the test measurement are given in table 1.

Table 1

station: Rwl. Delft				reference mark: Berkel			obs.: D. L. F. VAN LOON		
date	s	$\varphi = 51°59'$	σ_φ	$\lambda = -0^h17^m$	$\sigma_{\lambda \cos \varphi_0}$	$A = 92°53'$	σ_A	$A_g = 92°53'$	σ_{A_g}
March 26	8	55".189	0".45	$30^s.799$	$0^s.032$	24".609	0".65	24".501	0".19
March 27	8	53".577	0".35	$30^s.779$	$0^s.025$	23".800	0".49	23".945	0".15

Based on the above results, the instruments and the measuring programme were considered suitable for the measurements to be made in Ubachsberg and Tongeren.

6 Instrument set-up at Ubachsberg and Tongeren



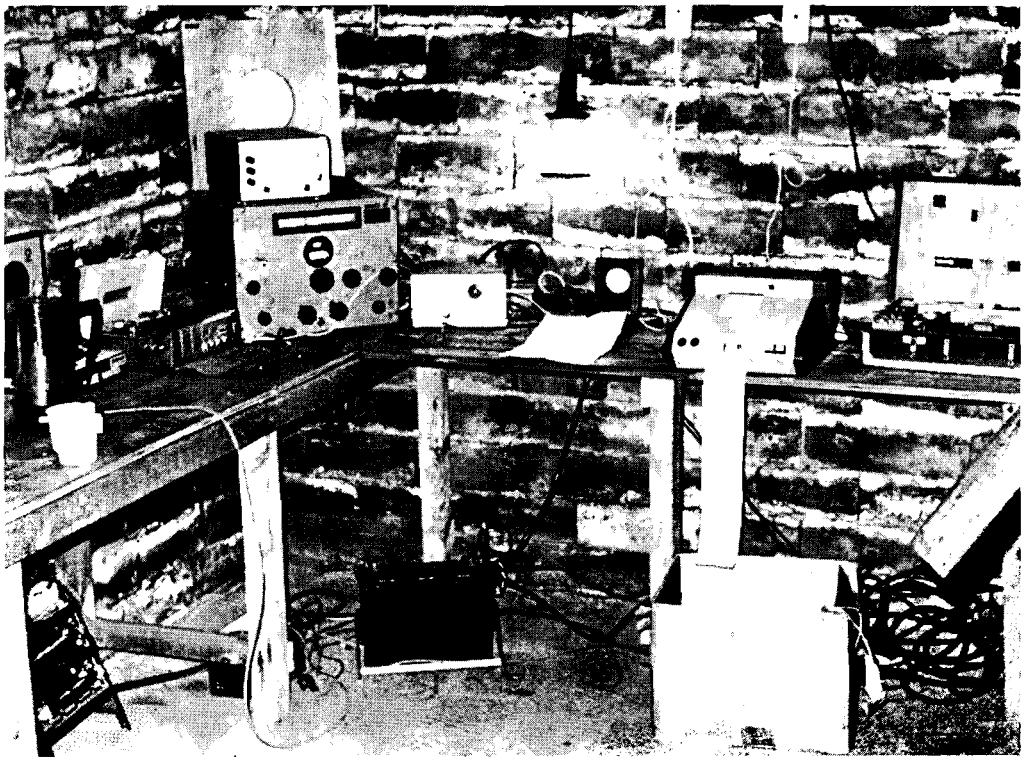
$e_1 = 8.254$ m
$\beta_1 = 234°28'49''$
$\alpha_1 = 9°13'11''.28$ $\sigma_{\alpha_1} = 0''.15$

$e_2 = 108.203$ m
$\beta_2 = 216°41'26''$
$\alpha_2 = 44°42'45''.98$ $\sigma_{\alpha_2} = 0''.15$

Fig. 1.

Ubachsberg. The original observation pillar, of 1890, was repaired in 1955 with the aid of the original reference mark. Because visibility from pillar '55 was not sufficient, a new concrete observation pillar '66 was built about 100 metres away from the old centre. The bearing and distance to the new centre and the angle α_2 between Tongeren (distance station) and Schimmert (reference mark) were measured. The time registration apparatus was mounted in a wooden shed near observation pillar '66.

Tongeren. In 1966 a new observation pillar was built on top of the spireless tower of the Basilica in the centre of the town, and the Military Geographic Institute in Brussels replaced the 1892 centre with the new centre, station A.



Set-up of time registration equipment at Tongeren

The bearing and distance to the new centre and the angle α_1, between Herderen (reference mark) and Ubachsberg (distance station), were measured again. The time registration apparatus was set up in a room situated under the flat roof in the church tower.

Contact between the observer and his assistant was maintained by means of an intercom.

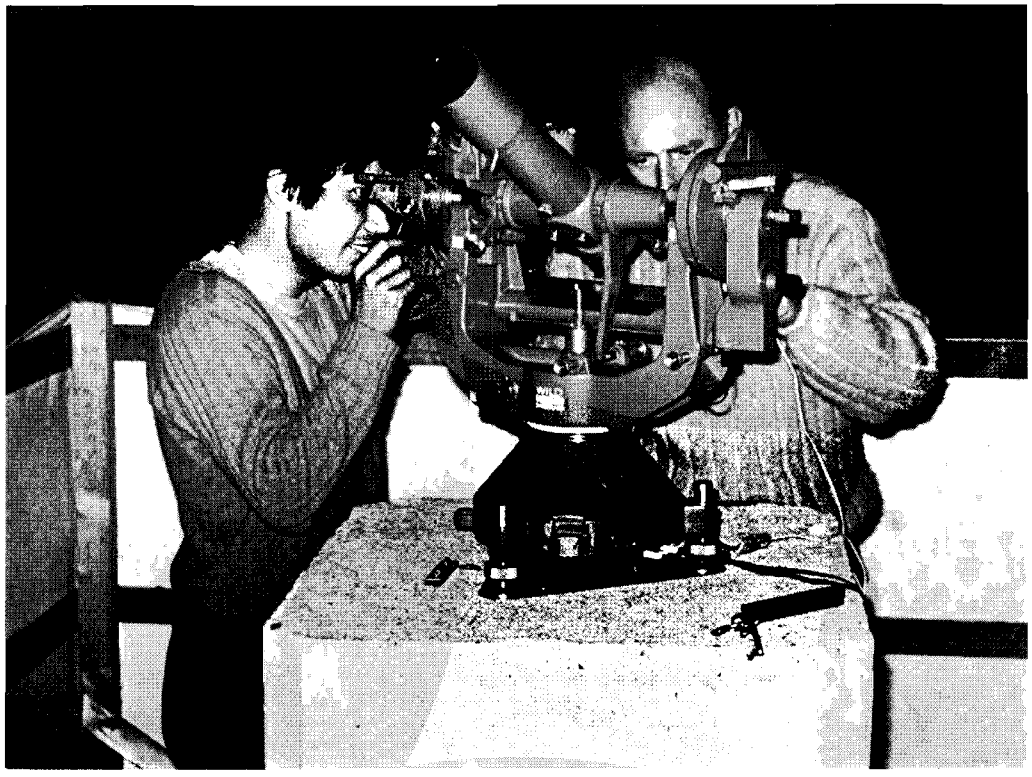
The Netherlands Triangulation Service computed plane rectangular coordinates of the new centres from the bearing and distance measurements to this centres. This coordinates and the geodetic positions are shown in table 2, in which the geodetic longitudes refer to the meridian of Amersfoort ($\lambda_{Am} = -5°23'15''.500$)

Table 2

station		x	y	φ_0	λ_0
Ubachsberg	Centre 1890	+39 845.615	−145 477.288	50°50'49".180	−0°33'56".926
	Pillar '66	+39 774.286	−145 558.665		
Tongeren	Centre 1892	+ 5 433.487	−152 902.284		
	Centre Stat. A	+ 5 433.945	−152 907.079	50°46'53".619	−0°04'37".393
	Pillar '66	+ 5 437.230	−152 899.507		

The angle of inclination of the line of sight was measured with the vertical circle of the T4, yielding the following results:

from Ubachsberg P'66 to Tongeren P'66: $H = -0°14'00''.6$
from Tongeren P'66 to Ubachsberg P'66: $H = -0°02'07''.8$



Star observation at Ubachsberg

The instruments remained permanently mounted at the stations for the duration of the measurements. A schematic diagram of the set-up is given in Fig. 2. When no observations were made, the T4 was protected against bad weather by a zinc cap.

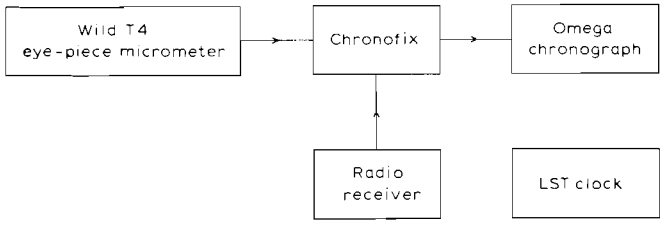


Fig. 2.

7 Measuring programme

The vertical axis of the Wild T4 was set vertically, as well as possible, using a suspension level. The suspension level was never reversed during the measurement. Star observations were conducted according to the following sequence:

- the reference mark was read twice in instrument position L;
- the telescope was set on the azimuth, and the zenith distance, ($z \simeq 60°$), on which the

Set-up of the Wild T4 at Tongeren

star to be observed was expected according to the star programme. The horizontal circle was read;

- as soon as the star appeared in the field of view, the suspension level was read; the telescope was adjusted with the vertical slow motion screw in such a way that the star passed as nearly as possible through the centre of the field of view; the star was tracked with the vertical wire over three complete revolutions of the eye-piece micrometer (= 27 contacts);
- immediately after this the telescope was set past the star with the slow motion screw, the suspension level was read and the star was tracked again with the vertical wire over three revolutions. After this the suspension level and the horizontal circle were read;
- the face of telescope was changed. After this, both the star and the reference mark were read twice.

The time signals were recorded on the chronograph with an accuracy of 1 millisecond. After

each star observation the position of the horizontal circle was moved by 22°.5 in order to eliminate systematic errors in the applied correction curve $R(\varphi)$. (Appendix 1).

8 Results

The input data for the computer were prepared in the field and sent by mail to the Geodetic Institute of the Delft University of Technology. In this way the provisional results were

Table 3

station: Tongeren, pillar '66				reference mark: Herderen			obs.: D. L. F. VAN LOON		
date 1968	s	$\varphi = 50°46'$	σ_φ	$\lambda = -0^h21^m$	$\sigma_{\lambda \cos \varphi_0}$	$A_g =$ 68°46′	σ_{A_g}	$A = 68°46'$	σ_A
May 14	8	56″.95	0″.57	51^s.189	0^s.040	29″.97	0″.24	30″.30	0″.77
May 23	8*	55 .90	0 .51	51 .221	0 .035	28 .80	0 .21	29 .50	0 .68
May 24	8	56 .74	0 .34	51 .286	0 .023	30 .40	0 .14	31 .85	0 .45
May 30	8	54 .50	0 .32	51 .301	0 .022	30 .81	0 .13	32 .46	0 .43
mean value		56 .02	0 .22	51 .249	0 .016	30 .00	0 .09	31 .03	0 .30
external			0 .48		0 .033		0 .20		0 .64

Remark: * the 8 stars were measured on different days: May 23: 3 stars, June 5: 3 stars and June 6: 2 stars.

Table 4

Station: Ubachsberg, pillar '66				reference mark: Schimmert			obs.: D. L. F. VAN LOON		
date 1968	s	$\varphi = 50°50'$	σ_φ	$\lambda = -0^h23^m$	$\sigma_{\lambda \cos \varphi_0}$	$A_g =$ 303°05′	σ_{A_g}	$A = 303°05'$	σ_A
June 11	4	53″.07	0″.70	47^s.899	0^s.045	5″.38	0″.28	4″.21	0″.87
June 14	8	50 .32	0 .46	48 .104	0 .029	5 .16	0 .18	6 .36	0 .57
June 18	4	51 .30	0 .20	48 .108	0 .012	5 .80	0 .08	7 .05	0 .24
June 21	8	50 .95	0 .23	48 .130	0 .016	5 .49	0 .09	7 .01	0 .30
June 26	4	50 .47	0 .93	47 .856	0 .065	5 .15	0 .39	3 .47	1 .23
July 3	8	50 .02	0 .36	48 .023	0 .024	4 .30	0 .15	4 .57	0 .47
mean value		50 .82	0 .20	48 .042	0 .013	5 .14	0 .08	5 .62	0 .25
external			0 .41		0 .027		0 .17		0 .53

known within a few days. The final computations were executed after the Bulletin Horaire had published the definite corrections. The final results are shown in tables 3 and 4.

The standard deviations were computed according to the formula (25). The mean values are obtained by attaching different weights to the results, depending on the number of stars. The internal accuracy of these mean values was derived from the above-mentioned standard deviations. Owing to the systematic errors between observations carried out on different nights (weather conditions, refraction, personal equation, etc.) these standard deviations are not very realistic. Therefore, the total of the observations per station was also adjusted, from which the external accuracy was computed according to formula (21). The ratio between the internal and the external accuracy is approximately a factor 2.

9 Reduction to centre

The reduction to centre of the mean values of the astronomical quantities (tables 3 and 4) was carried out using the measurement of bearing and distance to the centres concerned. The corrections were computed as follows:

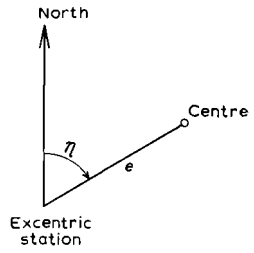


Fig. 3.

$$\left.\begin{aligned} \Delta\varphi'' &= \frac{\varrho'' e\cos\eta}{R} \\ \Delta\lambda^s &= \frac{-\varrho'' e\sin\eta}{15R\cos\varphi} \\ \Delta A'' &= \delta+\gamma \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (27)$$

in which δ is the bearing traverse and γ the convergence of meridians, according to:

$$\sin\delta = \frac{e_1\sin\beta_1 + e_2\sin\beta_2}{l} \quad \text{and} \quad \gamma = \frac{\varrho'' e\sin\eta\tan\varphi}{R} \quad \ldots\ldots\ldots\ldots \quad (28)$$

in which R is radius of the earth and l the distance between Tongeren and Ubachsberg. The results of these computations are given in tables 5 and 6 (see also Fig. 1).

From the astronomical quantities of table 5 and 6 the geodetic azimuth can be computed according to equation (1). It should be pointed out that only the standard deviation of the astronomical azimuths (see table 3 and 4) are affected by the reduction to centre, with regard to the standard deviation of the angle α_1 and α_2 ($\sigma_\alpha = 0''.15$), but the favourable negative correlation between the astronomical longitude and astronomical azimuth remained unchanged.

Table 5. Station Tongeren. Reduction to the centre

	φ		λ		A
Pillar '66	50°46′56″.020		−0h21m51s.249	Herderen	69°46′31″.030
$\Delta\varphi$	−0″.245	$\Delta\lambda$	+0s.011	$+\alpha_1$	9°13′11″.280
				Ubachsberg	77°59′42″.310
				$+\delta$	− 0°06′58″.222
				$+\gamma$	− 0″.130
Centre Stat. A	50°46′55″.775		−0h21m51s.238		77°52′43″.958

Table 6. Station Ubachsberg. Reduction to the centre

	φ		λ		A
Pillar '66	50°50′50″.820		−0h23m48s.042	Schimmert	303°05′05″.620
$\Delta\varphi$	+2″.612	$\Delta\lambda$	−0s.246	$-\alpha_2$	−44°42′45″.980
				Tongeren	258°22′19″.640
				$+\delta$	− 0°06′58″.222
				$+\gamma$	+ 2″.855
Centre 1890	50°50′53″.432		−0h23m48s.288		258°15′24″.273

10 Computation of the misclosure

From the results of tables 5 and 6, the misclosure between Ubachsberg and Tongeren is computed in table 7 according to [9]:

$$w_{k,i} = -\{(\lambda_k-\lambda_i)-(\lambda_k^g-\lambda_i^g)\}\sin\varphi_{k,i} + \{(A_{k,k'}-A_{i,i'})-(A_{k,k'}^g-A_{i,i'}^g)\} \quad \ldots\ldots\ldots \quad (29)$$

whereby the slight influence of the angle of inclination of the line of sight is omitted from consideration. In contrast with λ in preceding paragraphs, the longitude in formula (29) is positive east of Greenwich.

Table 7

k = Tongeren	i = Ubachsberg	k' = Ubachsberg	i' = Tongeren
λ_k	$0^h21^m51^s.238$	λ_k^g	0°04′37″.393
λ_i	$0^h23^m48^s.288$	λ_i^g	0°33′56″.926
	− 117s.050	$\lambda_k^g-\lambda_i^g$	−0°29′19″.533
$\lambda_k-\lambda_i$	−1755″.750		
$\lambda_k^g-\lambda_i^g$	−1759″.533	$\varphi_{k,i} = \frac{1}{2}(\varphi_k+\varphi_i)$	50°49′
difference	+ 3″.783	$\sin\varphi_{k,i}$	+0.7751
$A_{k,k'}$	77°52′43″.958	$A_{i,i'}$	258°15′24″.273
$A_{k,k'}^g$	77°52′42″.604	$A_{i,i'}^g$	258°15′26″.420
$A_{k,k'}-A_{k,k'}^g$	+ 1″.354		− 2″.147
$A_{i,i'}-A_{i,i'}^g$	− 2″.147		
difference	+ 3″.501		

From table 7 follows:

$$w_{k,i} = -(+3''.783)(+0.7751)+3''.501 = \boxed{+0''.57}$$

The misclosure with the twin Laplace station Leeuwarden-Ameland were computed in the same way, with the following results:

between Ubachsberg and Leeuwarden: $w = -2''.56$
between Tongeren and Leeuwarden: $w = -3''.10$

11 Standard deviation of the observations

Applying the law of propagation of variances to the quasi-observation (4) gives:

$$\sigma_l^2 = \sigma_a^2+\sigma_{\varphi_s}^2+\sigma_{\varphi_t}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (33)$$

The variance of the star's azimuth can be derived from the variance of the time observed by the eye-piece micrometer:

$$\sigma_a^2 = \sigma_T^2 \cos^2\delta \cos^2 q \operatorname{cosec}^2 z \quad \ldots \quad (34)$$

in which σ_T^2 was determined by comparing two series of times obtained from two successive trackings of one star at the same position of the instrument. The results of this computation are shown in Appendix 4, indicating clearly that σ_T^2 correlates with the horizontal component of the star's velocity. A straight line:

$$\sigma_T^2 = 98 + 167 \sec^2\delta \sec^2 q \quad \text{in msec}^2 \quad \ldots \quad (35)$$

computed through the plotted points represents the average variance of time, corresponding to the mean value of 27 contacts. Substitution of (35) into (34) gives:

$$\sigma_a^2 = (167 + 98 \cos^2\delta \cos^2 q) \operatorname{cosec}^2 z \quad \ldots \quad (36)$$

Considering two stars with extreme horizontal velocity components:

$$v_H = 15''/\text{sec}: \qquad \sigma_a = 0''.24 \operatorname{cosec} z$$

$$v_H = 5.5''/\text{sec}: \qquad \sigma_a = 0''.20 \operatorname{cosec} z$$

it seems reasonable to assume an average value for the computed azimuth of the star:

$$\sigma_a = 0''.22 \operatorname{cosec} z \quad \ldots \quad (37)$$

The variance $\sigma_{\varphi_s}^2$ follows from (11):

$$\sigma_{\varphi_s}^2 = \sigma_{\varphi_s'}^2 + \sigma_{R(\varphi)}^2 + p^2 \cot^2 z \, \sigma_M^2 = 0''.25^2 \quad \ldots \quad (38)$$

in which:

$\sigma_{\varphi_s'} = 0''.22$ (standard deviation of the circle reading, including the standard deviation of the circle division)

$\sigma_{R(\varphi)} = 0''.04$ (standard deviation of the correction for systematic circle error)

$\sigma_M = 0''.20\sqrt{p}$ (standard deviation of the reading of the suspension level)

The variance $\sigma_{\varphi_t}^2$ follows from (12):

$$\sigma_{\varphi_t}^2 = \sigma_{\varphi_t'}^2 + \sigma_{R(\varphi)}^2 = 0''.36^2 \quad \ldots \quad (39)$$

where:

$\sigma_{\varphi_t'} = 0''.35$ (standard deviation of pointing to the reference mark, including the standard deviation of the circle reading and the standard deviation of the circle division)

Substitution of (37), (38) and (39) into (33) gives:

$$\boxed{\sigma_l = 0''.51} \qquad (40)$$

This value can also be derived directly from the quasi-observations by computing the differences:

$$d = l_1 - l_2 \qquad \text{and} \qquad d = l_3 - l_4$$

for each star. For all the 68 star observations in Ubachsberg and Tongeren we obtain:

$$\sigma_l^2 = \frac{[d^2]}{2n} = \frac{95.10}{2 \times 2 \times 68} \rightarrow \boxed{\sigma_l = 0''.59} \qquad (41)$$

This is very close to the value 0''.51 in (40) which was obtained from an integration of the random errors. The small difference, caused by some external factors, has little significance.

On the other hand we have:

$$Q = \begin{pmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 0.25 \end{pmatrix} \qquad (42)$$

which are the weight coefficients for the quantities φ, $\lambda \cos \varphi_0$, and A_g, referring to 4 arbitrary stars regularly distributed in azimuth at $z = 60°$. Assuming that each star is observed in 4 series, the standard deviations to be expected follows from (41) and (42) according to (25)

		contribution per star:
σ_φ	$= \frac{0.59}{4}\sqrt{1.5} = 0''.36$	$0''.72$
$\sigma_{\lambda \cos \varphi_0}$	$= \frac{0.59}{4}\sqrt{1.5} = 0''.36$	$0''.72$
σ_A	$= \frac{0.59}{4}\sqrt{2.52} = 0''.47$	$0''.94$
σ_{Ag}	$= \frac{0.59}{4}\sqrt{0.25} = 0''.15$	$0''.30$

(43)

The contribution per star can also be computed from the external accuracy obtained in tables 3 and 4, by multiplying these values by $\sqrt{32}$ and $\sqrt{36}$ (the total number of stars at each station) respectively. These results are shown in table 8.

Table 8

	Tongeren	Ubachsberg	contribution per star		ratio
			external (mean)	internal	
σ_φ	2″.66	2″.40	2″.53	0″.72	~3.5
$\sigma_{\lambda \cos \varphi_0}$	2 .80	2 .43	2 .61	0 .72	
σ_A	3 .62	3 .12	3 .37	0 .94	
σ_{A_g}	1 .13	0 .96	1 .05	0 .30	

Comparing the external and internal accuracy in this way, we obtain a ratio of approximately 3.5, due obviously to the influence of various external factors, such as lateral refraction, random and systematic errors in the star's coordinates, and variation in the personal equation of the observer.

12 Standard deviation of the geodetic part of the Laplace equation

The misclosure between two Laplace stations was computed according to formula (26). It is clear that the standard deviation of the geodetic quantities in this equation also contributes to the standard deviation of the misclosure. Prof. ROELOFS suggested investigation of the precision of the geodetic part of the Laplace equation:

$$L = -(A_{k,k'} - A_{i,i'}) + (\lambda_k - \lambda_i) \sin \varphi_{k,i}$$

in order to estimate the required precision of the astronomical part. The computation, carried out by Ir. J. C. P. DE KRUIF of the Computer Centre of the Geodetic Department of the Delft University of Technology, yields the following weight coefficients:

	1	2	3
1	+0.494	−0.221	−0.273
2	−0.221	+0.477	−0.257
3	−0.273	−0.257	+0.530

Taking the square root of these values and multiplying by $\sigma = 1''$, we obtain:

		σ_L
1 =	Zierikzee – Leeuwarden	0″.70
2 =	Ubachsberg – Zierikzee	0″.69
3 =	Leeuwarden – Ubachsberg	0″.73

These values are surprisingly small, owing to a negative correlation effect in the propagation of the variances.

It is difficult to establish exact requirements for the accuracy of the astronomical part of the Laplace quantity, but the requirement "the highest possible accuracy" may be replaced

by the demand "an insignificant contribution to the variance of the misclosure". In this sense a value of 0".25 for the standard deviation of the mean value of the geodetic azimuth can be considered as satisfactory: its contribution is very small:

$$\sigma_w^2 = 0''.70^2 + 0''.25^2 = 0''.74^2$$

Considering the standard deviation of the Laplace stations

$$\left.\begin{array}{ll} \text{Leeuwarden – Ameland:} & \sigma_{A_g} = 0''.16 \\ \text{Ubachsberg – Tongeren:} & \sigma_{A_g} = 0''.17 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (44)$$

it can be concluded that these results are quite acceptable.

13 Personal equation of the observer

Immediately after the measurements in Tongeren and Ubachsberg, the personal equation of the observer was determined using an artificial moving star in a laboratory room of the Geodetic Department of the Delft University of Technology. The observations were made on several successive days in a close approximation of the way in which the stars in the field had been observed. The results of this investigation are mentioned below.

The variances of the time determination are plotted against the horizontal velocity component in Appendix 5, yielding the following relation:

$$\boxed{\sigma_T^2 = 46 + 65 \sec^2\delta \sec^2 q} \quad \text{in msec}^2 \quad \text{(from } N = 27 \text{ contacts)} \ldots\ldots \quad (45)$$

This is about a factor 1.5 better than the results obtained from the star observations shown in Appendix 4, which can be explained by the much more favourable conditions in the laboratory.

In Appendix 6 the corrections for the personal equation of Mr. D. L. F. VAN LOON, (who executed all the observations in Tongeren and Ubachsberg) are represented in a diagram. The observations on the various days are indicated by different symbols. Each symbol represents the average of the personal equation derived from 4×27 contact times. It shows clearly that the personal equation and the horizontal velocity component of the star are correlated. The average correction for the personal equation is:

$$\boxed{P = -47 + 18 \sec\delta \sec q} \quad \text{in msec (D. L. F. v. Loon)} \ldots\ldots\ldots \quad (46)$$

The personal equation of Mr. H. A. VERHOEF and Mr. C. DE VRIES, both very experienced observers, are given in Appendix 7 and 8, revealing that the characteristics of the personal equation per observer can be very different:

$$P = -39 - 3 \sec\delta \sec q \quad \text{in msec} \quad \text{(H. A. Verhoef)} \ldots\ldots\ldots\ldots \quad (47)$$

$$P = +18 - 52 \sec\delta \sec q \quad \text{in msec} \quad \text{(C. de Vries)} \ldots\ldots\ldots\ldots \quad (48)$$

The observer's personal equation was not included in the final computations as the artificial star observations were made after the completion of the measurements at Ubachsberg and Tongeren and thus provide no control over the change in his personal equation during the period of the mesaurements.

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[3] J. C. DE MUNCK – Recherche sur la détermination précise du profil des axes de rotation. Report I.A.G. General Assembly, Rome, 1954.
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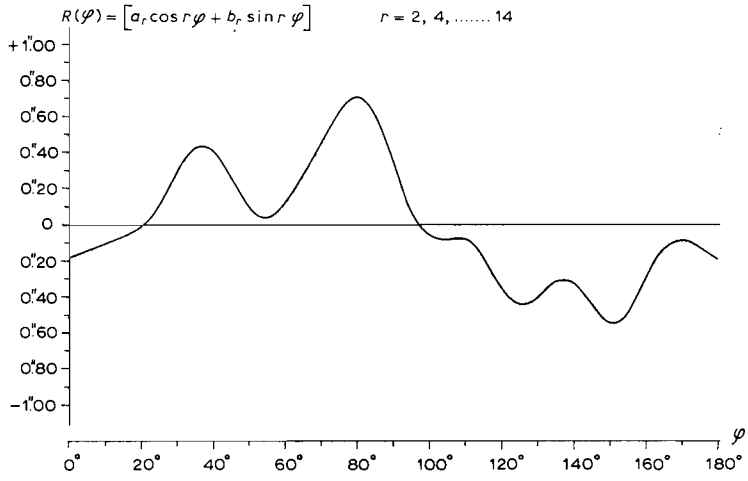
[8] A. R. ROBBINS – Time in geodetic astronomy. Surv. Rev., January, 1967.
[9] W. BAARDA – Some remarks on the computation and adjustment of large system of geodetic triangulation. Bull. Géod. 1957, No. 3.

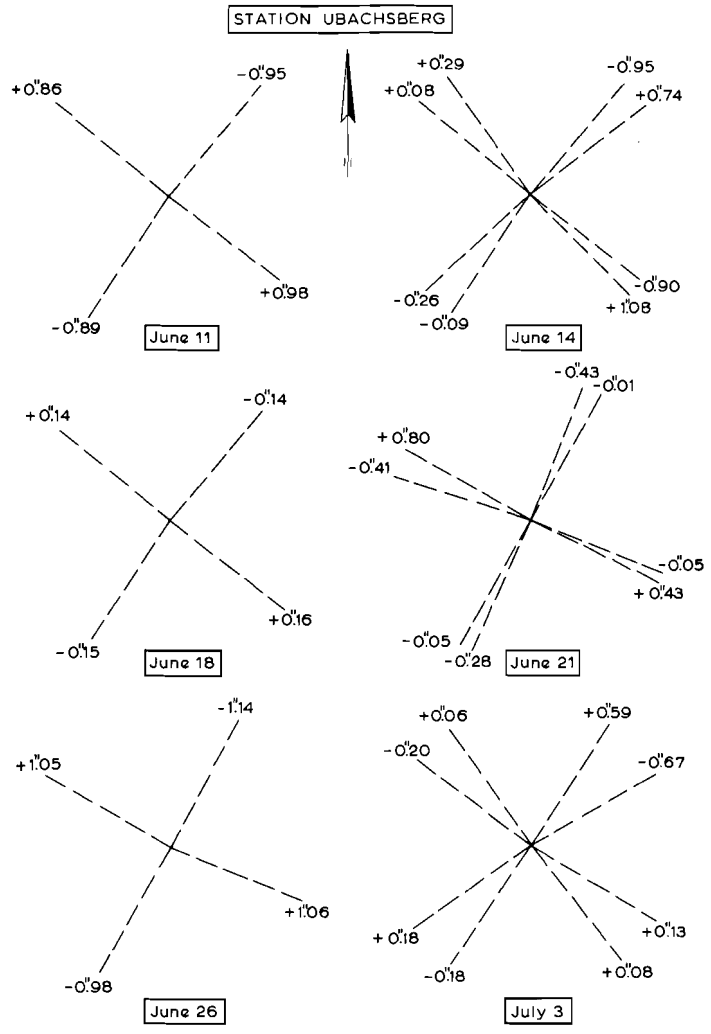
APPENDIXES



a_2	= −0".196	b_2	= +0".347
a_4	= +0.108	b_4	= −0.039
a_6	= −0.081	b_6	= +0.033
a_8	= +0.026	b_8	= −0.154
a_{10}	= +0.065	b_{10}	= +0.015
a_{12}	= −0.052	b_{12}	= −0.030
a_{14}	= −0.045	b_{14}	= +0.046

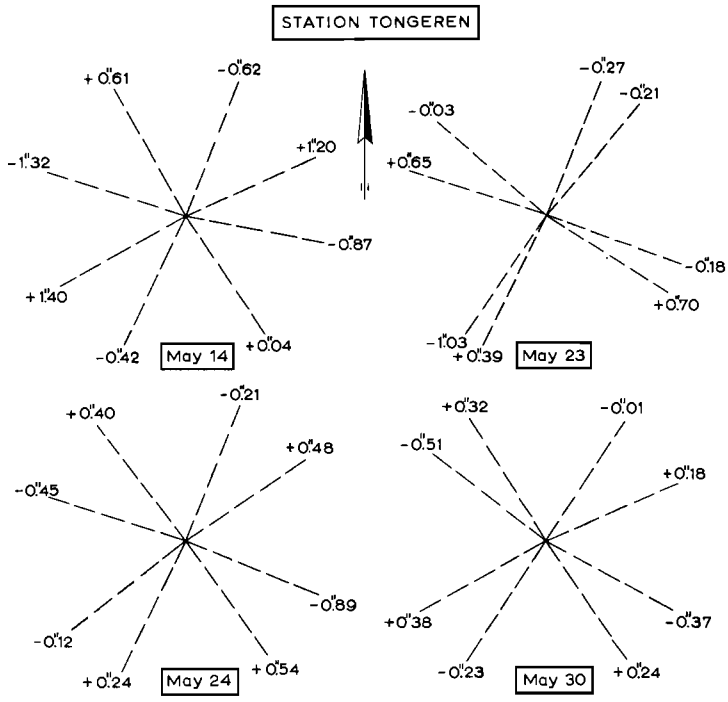
SYSTEMATIC ERRORS IN THE HORIZONTAL CIRCLE DIVISION
OF THE UNIVERSAL THEODOLITE WILD T4

Appendix 1



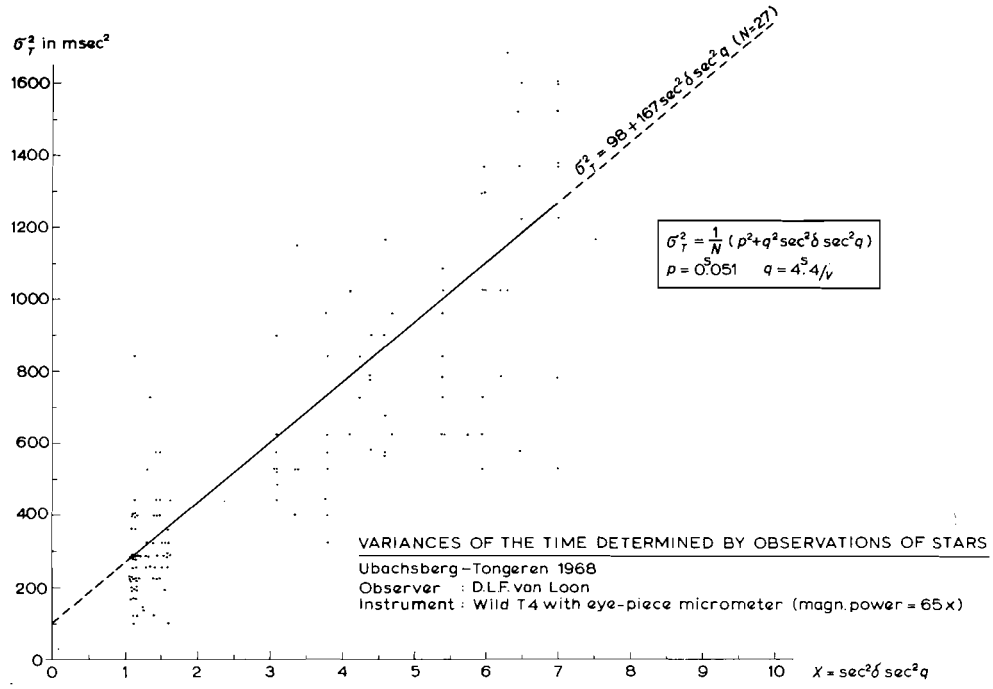
THE SELECTED STARS AND THE CORRECTIONS $\bar{v}_i$ TO THE QUASI-OBSERVATIONS

Appendix 2

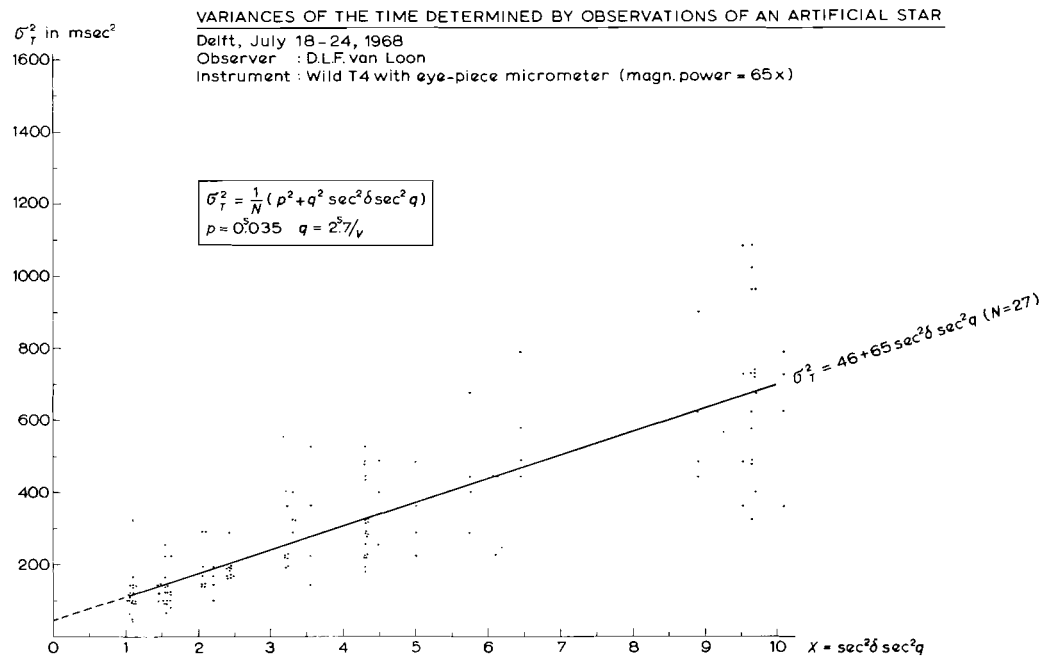


THE SELECTED STARS AND THE CORRECTIONS $\bar{v}_i$ TO THE QUASI-OBSERVATIONS

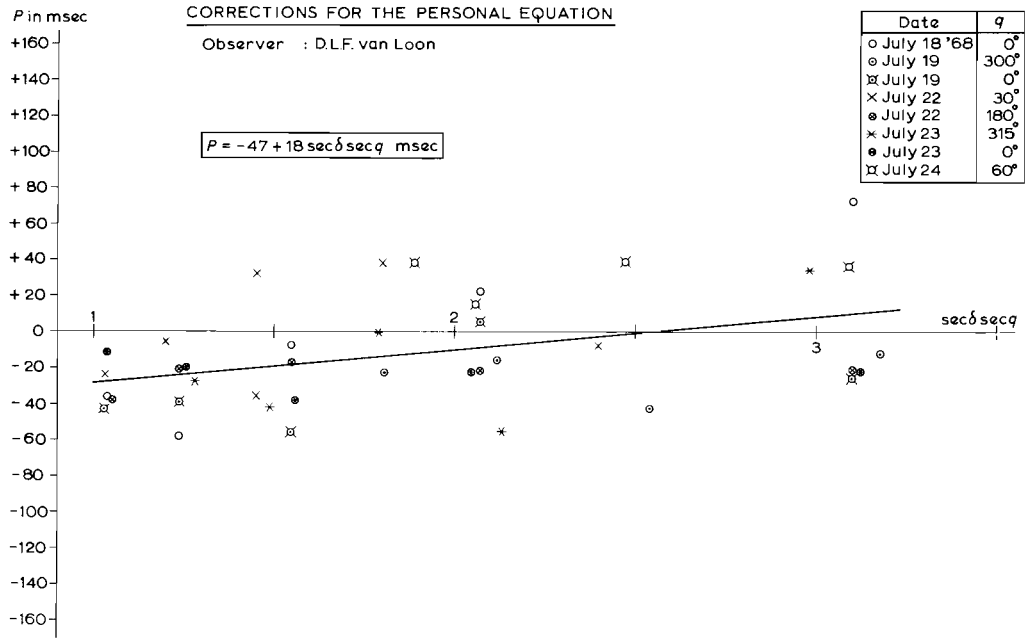
Appendix 3



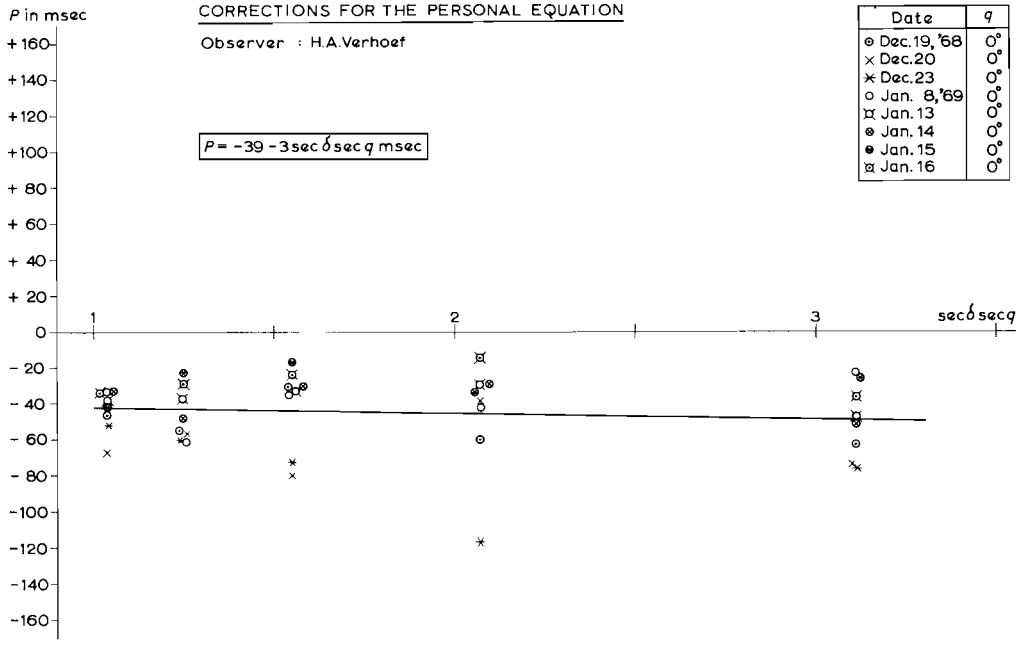
Appendix 4



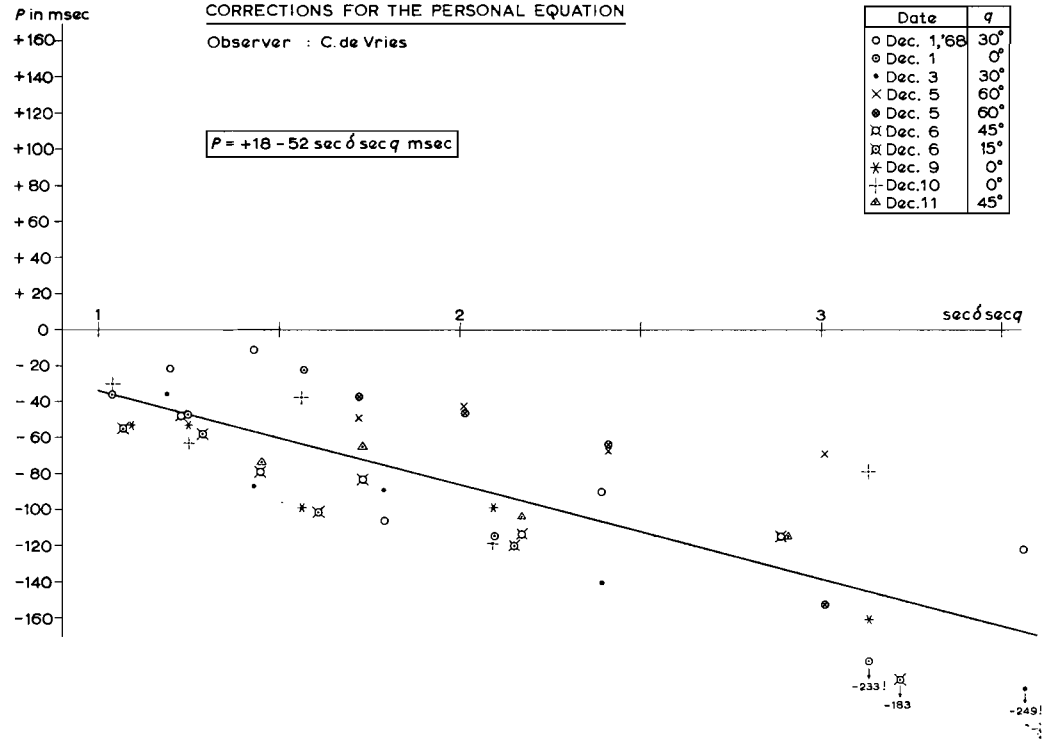
Appendix 5



Appendix 6



Appendix 7


P in msec
CORRECTIONS FOR THE PERSONAL EQUATION
Observer : C. de Vries
P = +18 - 52 sec ó sec q msec
Date
q
o Dec. 1,'68 30°
⊙ Dec. 1 0°
• Dec. 3 30°
× Dec. 5 60°
⊛ Dec. 5 60°
⊠ Dec. 6 45°
⊡ Dec. 6 15°
✳ Dec. 9 0°
⊹ Dec.10 0°
△ Dec.11 45°
+160
+140
+120
+100
+ 80
+ 60
+ 40
+ 20
0
- 20
- 40
- 60
- 80
-100
-120
-140
-160
1
2
3
sec ó sec q
-233!
-183
-249!

Appendix 8