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THE ASTROMETRIC PROCEDURE OF SATELLITE PLATE REDUCTION AS APPLIED AT THE DELFT GEODETIC INSTITUTE

A description with some results for
WEST, NGSP and ISAGEX

by

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SUMMARY

In 1966, following some years of preparation, the Delft Working Group for Satellite Geodesy started photographic observations of satellites. Since then the camera station of the Geodetic Institute of Delft University of Technology has continued participating in internationally coordinated geodetic satellite observation programmes. Contributions were made to the Western European Satellite Triangulation Programme (WEST), the National Geodetic Satellites Programme (NGSP), the International Satellite Geodesy Experiment (ISAGEX), and a Short Arc Observation Programme. Both optically passive and optically active satellites were observed. In 1969 the station was relocated, but still it remained in the vicinity of Delft until definitive re-establishment followed in 1973 at a more suitable site near Apeldoorn [1].

A previous publication [2] dealt in particular with the equipment in use for photographic observation of station-to-satellite directions. The present publication concentrates on the formulas applied for the reduction of observations made with the Delft TA-120 camera in three of the four mentioned programmes. The curve fitting procedure to assess the quality of the observations is also described and finally results of observations and computations are given.

THE ASTROMETRIC PROCEDURE OF SATELLITE PLATE REDUCTION AS APPLIED AT THE DELFT GEODETIC INSTITUTE

1 Introduction

Since 1st August, 1966, the Working Group for Satellite Geodesy has participated in a number of international observation programmes. In this publication special attention will be given to the system of reduction formulas by means of which the initial information of the photographs has been transformed into relevant geodetic data. The photographs referred to are those taken with the equatorially mounted TA-120 concentric-mirror type camera (Bouwers-Maksutov).

All observations made in the WEST-programme, NGSP and ISAGEX are treated with this same system of reduction formulas, to be discussed here. Observations made in connection with the current European Short Arc Programme are reduced by means of a modified version of the procedure to be described. The authors intend to indicate these modifications in a later publication.

The observations have been made from two different observing-sites:

- DELFT, WIPPOLDER (Fig. 1) until 1st December, 1969, and
- DELFT, YPENBURG (Fig. 2) from 1st December 1969 until 1st December, 1973.

For the correct location data, see [3].

The present publication must be understood as an account of work accomplished during the period 1966–1971. Photographic observations of satellites for geodesy are being continued from a recently established observatory near Apeldoorn [1], partly with new equipment [2]. The material presented can have only marginal scientific interest, because the photographic technique of satellite observations for high precision geodesy has largely lost its importance. Moreover the computing procedures outlined are to a great extent standard.

2 Some technical details

An astrometric (short-Turner) method is applied to reduce stellar oriented photographic satellite plates to fixed-earth station-to-satellite directions. The plates considered are those taken with the TA-120 camera in use by the Delft Geodetic Institute.

The relevant optical features of the TA-120 camera are:

optics:	Bouwers-Maksutov concentric mirror
focal length:	120 cm
effective aperture:	21 cm
field:	$5^\circ \times 5^\circ$ spherical.

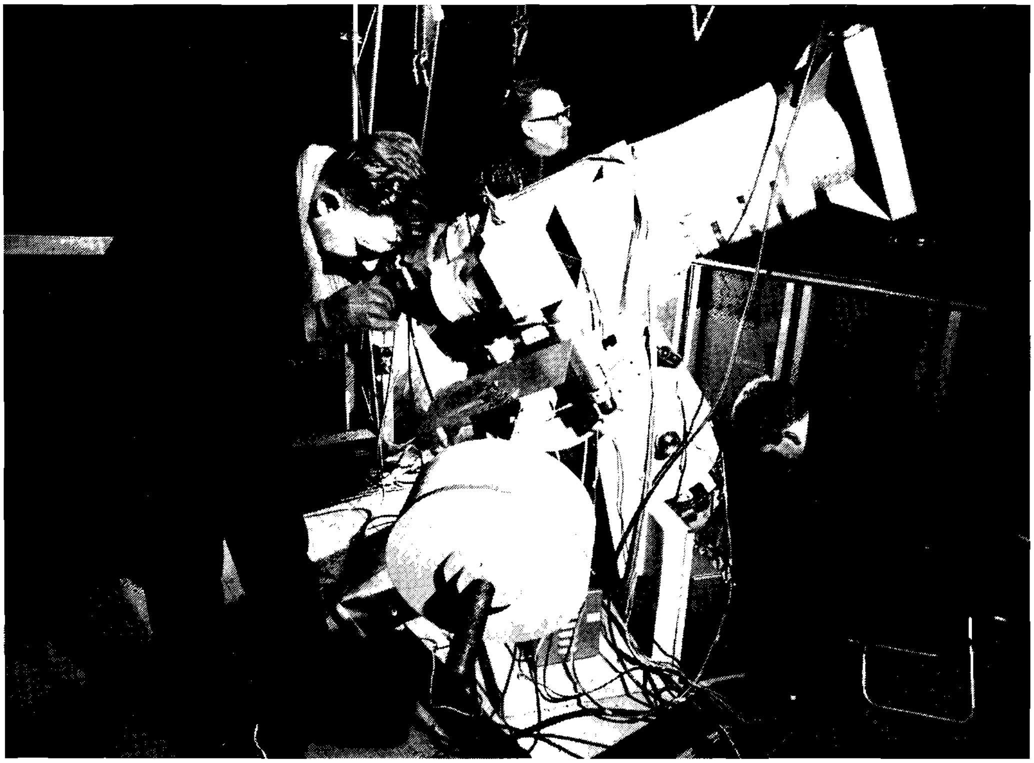


Fig. 1. The Bouwers-Maksutov camera in the original mount at the Wippolder-site.

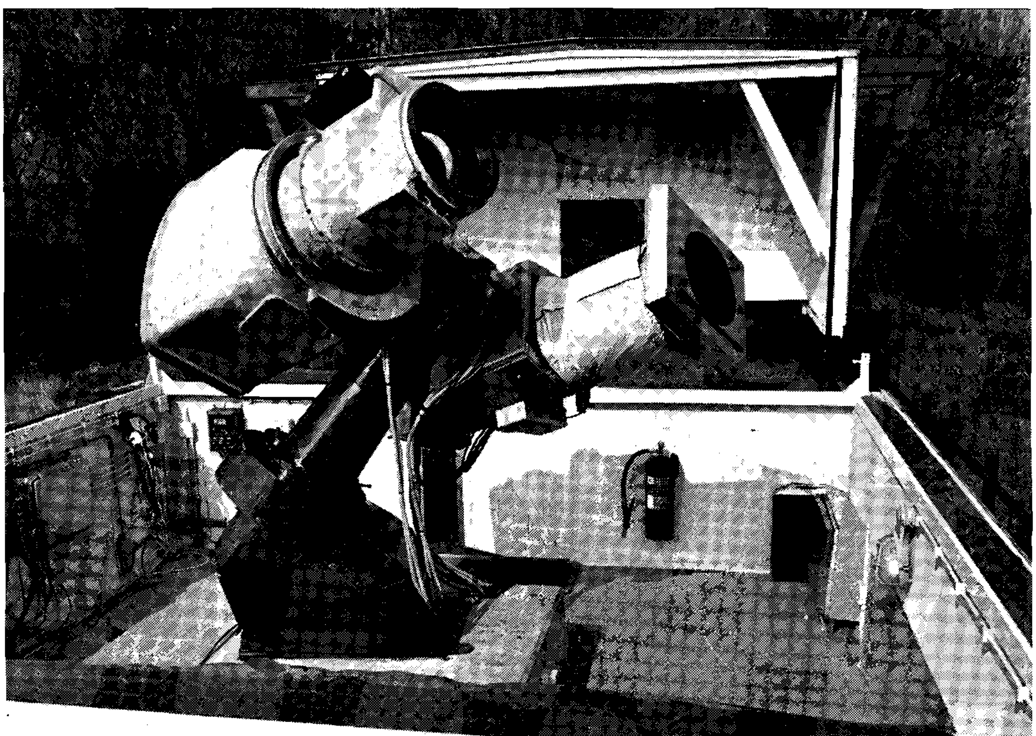


Fig. 2. The Bouwers-Maksutov- and the K-50 camera in the Rademakers Minimount at the Ypenburg-site.

The camera operates on 5 inches wide roll-film (currently Kodak 2475 Estar Base), each frame before exposure being pressed to assume a spherical shape with about 240 cm radius of curvature.

The camera is mounted equatorially and driven at the sidereal rate.

With optically passive satellites, timing of satellite images is achieved by means of a focal plane chopper of special design. The essential point here is that during the exposure of the satellite trail two narrow strips of light weight material (about 2 cm wide), mutually seperated by about 0.5 cm between the strips, periodically chop the light beam from the satellite just before it could reach the focal plane. This produces each time a satellite image in the center of a trail interruption. Time control of the chopper is obtained by recording photo-electronically the instants that the twin-strips assume four selected and equally spaced calibration positions, known geometrically with respect to the camera's fiducial marks. Finally numerical interpolation yields for each satellite image the time instant at which the chopper occupied the position in which it produced that image.

Each successful film frame is copied onto a glass plate, which is subsequently measured on a Mann 422F XY-comparator. A plate is measured in two positions, mutually rotated over about 180°. For each satellite image six reference stars are selected, evenly distributed with respect to the satellite image and as close to it as is practical. Throughout the measurements and the subsequent calculations each satellite image with its reference stars is treated individually. However it proved inevitable to assign identical reference stars to adjacent satellite images. The sequence of measurement is as follows: satellite image-reference stars-reference stars in reversed order-satellite image. This cycle is repeated once, before the operator proceeds to the next satellite image. When all satellite images have been treated in this way the entire procedure is performed with the plate rotated through about 180°.

Reference star positions are taken from a magnetic tape version of the SAO star catalog by means of a computer search programme.

The computational part of the plate reduction is performed in three computer programmes briefly indicated by:

1. "Time reduction"
2. "Position reduction"
3. "Conversion to fixed-Earth reference".

In the following three sections these programmes are described consecutively, giving details of the formula's used.

3 Time reduction

This programme reduces the chopper-time records to time instants related to the satellite positions recorded on the photographic plate. Corrections are applied for receiver delay, propagation time of the 75 kHz HBG time signal from Neuchâtel to Delft and for the time difference between *UTC* and the HBG emission.

Suppose the chopper traverses the rectangular coordinate system defined by the fiducial marks in x-direction and specify the calibration positions by x^1, x^2, x^3, x^4 respectively.

Denote the instants recorded for these positions and for satellite image sub. i (approximate coordinates x_i, y_i) by respectively:

$$t_0+\Delta t_i^1, t_0+\Delta t_i^2, t_0+\Delta t_i^3, t_0+\Delta t_i^4$$

Then, disregarding receiver delay and emission and propagation corrections, a provisional time instant $\bar{t}_i$ for the recording of image sub. i is obtained from:

$$\bar{t}_i = t_0+\Delta t_i \qquad (3.1)$$

in which:

$$\Delta t_i = ((x_i)^2, x_i, 1)\cdot \underline{a} \qquad (3.2)$$

with:

$$\underline{a} = (M^{*}\cdot M)^{-1}\cdot M^{*}\cdot \underline{t} \qquad (3.3)$$

if:

$$M = \begin{pmatrix} (x^1)^2 & x^1 & 1 \\ (x^2)^2 & x^2 & 1 \\ (x^3)^2 & x^3 & 1 \\ (x^4)^2 & x^4 & 1 \end{pmatrix} \quad \text{and} \quad \underline{t} = \begin{pmatrix} \Delta t_i^1 \\ \Delta t_i^2 \\ \Delta t_i^3 \\ \Delta t_i^4 \end{pmatrix} \qquad (3.4)$$

Until 1st May 1972 a local time standard was by means of a variable delay brought in temporary synchronism with the received HBG time signals, just before a satellite observation.

Hence until that date, in order to relate the satellite image recording instants to *UTC*, $\bar{t}_i$ had to be corrected as follows:

$$t_i = \bar{t}_i+\Delta_d+\Delta_p+E \qquad (3.5)$$

in which:

Δ_d = receiver delay = 1.5 ms
Δ_p = propagation correction = 2.2 ms
E = "*UTC*−signal" as published in circular D by BIH.

Since 1st May 1972 a rubidium time and frequency standard (HP 5065 A) is used to keep *UTC* between periodic flying clock visits. This technique meets the needs of satellite photography to an extent that corrections from $\bar{t}_i$ to t_i could be omitted since that date.

4 Position reduction

This programme reduces plate measurements of satellite and star images to provisional topocentric geometric satellite directions referred to the astrometric system adopted for the SAO catalog (equinox 1950.0, system FK4). The directions are provisional in that no corrections will be applied for annual aberration, diurnal aberration, light travel time, parallactic refraction and satellite phase.

Denote plate measurement positions by I and II respectively.

Denote arithmetic means over all four satellite and star image measurements expressed in mm and after division by the focal length (1200 mm) as follows:

satellite image sub. i:

$$\bar{X}_i^{\mathrm{I}}, \bar{Y}_i^{\mathrm{I}}; \qquad \bar{X}_i^{\mathrm{II}}, \bar{Y}_i^{\mathrm{II}}$$

---

* Indicating transposition.

image star sub. k as related to satellite image sub. i:

$$\bar{x}^{\mathrm{I}}_{i,k},\ \bar{y}^{\mathrm{I}}_{i,k};\qquad \bar{x}^{\mathrm{II}}_{i,k},\ \bar{y}^{\mathrm{II}}_{i,k}$$

If MJD is the Modified Julian Date of observation (integer number), then stellar positions updated for proper motion are:

$$\left.\begin{aligned}\alpha_k &= \alpha_{1950,k}+\tau\mu_k\\ \delta_k &= \delta_{1950,k}+\tau\mu'_k\end{aligned}\right\} \qquad (4.1)$$

where (omitting subscript k) α_{1950}, δ_{1950} and μ, μ' are taken from the SAO catalog, and:

$$\tau = \frac{MJD-33282}{365.24} \qquad (4.2)$$

Adopt approximate right ascension A_i and approximate declination D_i for the direction associated with satellite image sub. i.

Then solve standard coordinates $\xi_{i,k}$, $\eta_{i,k}$ from:

$$\begin{pmatrix}\cos\eta_{i,k}\cos\xi_{i,k}\\ \cos\eta_{i,k}\sin\xi_{i,k}\\ \sin\eta_{i,k}\end{pmatrix} = T_i\cdot\begin{pmatrix}\cos\delta_k\cos\alpha_k\\ \cos\delta_k\sin\alpha_k\\ \sin\delta_k\end{pmatrix} \qquad (4.3)$$

with:

$$T_i = \begin{pmatrix}\cos D_i\cos A_i & \cos D_i\sin A_i & \sin D_i\\ -\sin A_i & \cos A_i & 0\\ -\sin D_i\cos A_i & -\sin D_i\sin A_i & \cos D_i\end{pmatrix} \qquad (4.4)$$

Now, for both plate measurement positions, form:

$$\underline{l}_i = \begin{bmatrix}\vdots\\ \xi_{i,k}-\bar{x}_{i,k}\\ \vdots\\ \eta_{i,k}-\bar{y}_{i,k}\\ \vdots\end{bmatrix} \qquad (4.5)$$

and:

$$M_i = \left[\begin{array}{ccc|ccc}\vdots & \vdots & \vdots & & & \\ \bar{x}_{i,k} & \bar{y}_{i,k} & 1 & & 0 & \\ \vdots & \vdots & \vdots & & & \\ \hline & & & \vdots & \vdots & \vdots\\ & 0 & & \bar{x}_{i,k} & \bar{y}_{i,k} & 1\\ & & & \vdots & \vdots & \vdots\end{array}\right] \qquad (4.6)$$

Then, assuming a Gaussian (normal) probability distribution for the components of $\underline{l}_i$, with correlation freedom and constant variance, the most probable $\underline{a}_i$ of linear plate constants is obtained independently for both plate measurement positions:

$$\underline{a}_i = V\{\underline{a}_i\} \cdot M_i^* \cdot \underline{l}_i \qquad (4.7)$$

with:

$$V\{\underline{a}_i\} = (M_i^* \cdot M_i)^{-1} \qquad (4.8)$$

The standard coordinates for the station-to-satellite direction become:

$$\begin{pmatrix} \xi_i \\ \eta_i \end{pmatrix} = B_i \cdot \underline{a}_i + \begin{pmatrix} \bar{X}_i \\ \bar{Y}_i \end{pmatrix} \qquad (4.9)$$

for both plate measurement positions independently, if:

$$B_i = \begin{pmatrix} \bar{X}_i & \bar{Y}_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{X}_i & \bar{Y}_i & 1 \end{pmatrix} \qquad (4.10)$$

Unit weight variance in (seconds of arc)^2 is estimated from:

$$\hat{\sigma}_i^2 = \frac{\underline{v}_i^* \cdot \underline{v}_i}{2s_i - 6} \cdot (206265)^2 \qquad (4.11)$$

where s_i is the number of reference stars used (usually six) and the correction vector $\underline{v}_i$ is obtained from:

$$\underline{v}_i = \underline{l}_i - M_i \cdot \underline{a}_i \qquad (4.12)$$

Standard coordinates from both plate measurement positions are combined to mean values:

$$\xi_i = \frac{\xi_i^{\mathrm{I}} + \xi_i^{\mathrm{II}}}{2}; \qquad \eta_i = \frac{\eta_i^{\mathrm{I}} + \eta_i^{\mathrm{II}}}{2} \qquad (4.13)$$

These are transformed into right ascension and declination by solving α_i, δ_i from:

$$\begin{pmatrix} \cos\delta_i \cos\alpha_i \\ \cos\delta_i \sin\alpha_i \\ \sin\delta_i \end{pmatrix} = T_i^* \cdot \begin{pmatrix} \cos\eta_i \cos\xi_i \\ \cos\eta_i \sin\xi_i \\ \sin\eta_i \end{pmatrix} \qquad (4.14)$$

Now suppose the satellite trail makes an angle ψ with the positive comparator Y-axis and moreover suppose that along trail comparator measurements have a standard deviation g times that of across trail measurements, then the variance-covariance matrix of the mean standard coordinates will be:

$$V\begin{Bmatrix} \xi_i \\ \eta_i \end{Bmatrix} = \tfrac{1}{4}(\hat{\sigma}_i^{\mathrm{I}})^2 B_i^{\mathrm{I}} \cdot V\{\underline{a}_i^{\mathrm{I}}\} \cdot (B_i^{\mathrm{I}})^* + \tfrac{1}{4}(\hat{\sigma}_i^{\mathrm{II}})^2 B_i^{\mathrm{II}} \cdot V\{\underline{a}_i^{\mathrm{II}}\} \cdot (B_i^{\mathrm{II}})^* + \tfrac{1}{2}\sigma^2 R \cdot \begin{pmatrix} g^2 & 0 \\ 0 & 1 \end{pmatrix} \cdot R^* \qquad (4.15)$$

if:

$$R = \begin{pmatrix} \sin\psi & -\cos\psi \\ \cos\psi & \sin\psi \end{pmatrix} \quad \ldots\ldots \quad (4.16)$$

and σ is the standard deviation of across trail comparator measurements, expressed in seconds of arc.

Defining

$$E = \begin{pmatrix} \sec D_i & 0 \\ 0 & 1 \end{pmatrix} \quad \ldots\ldots \quad (4.17)$$

the variance-covariance matrix of α_i, δ_i becomes:

$$V\begin{Bmatrix} \alpha_i \\ \delta_i \end{Bmatrix} = \begin{pmatrix} \sigma^2_{\alpha_i} & \sigma_{\alpha_i\delta_i} \\ \sigma_{\delta_i\alpha_i} & \sigma^2_{\delta_i} \end{pmatrix} = E \cdot V\begin{Bmatrix} \xi_i \\ \eta_i \end{Bmatrix} \cdot E \quad \ldots\ldots \quad (4.18)$$

Because of the simplifying assumptions as regards the statistical properties of the components of $\underline{l}_i$, the off-diagonal elements of both

$$V\begin{Bmatrix} \xi_i \\ \eta_i \end{Bmatrix} \quad \text{and} \quad V\begin{Bmatrix} \alpha_i \\ \delta_i \end{Bmatrix}$$

should be zero.

The essential output of this programme consists of

$$\alpha_i, \delta_i \quad \text{and} \quad \sigma_{\alpha_i}, \sigma_{\delta_i}$$

α_i, δ_i should be interpreted as is done in the beginning of this section.

5 Conversion to fixed-Earth reference

This programme transforms the station-to-satellite directions as derived in the previous section to a fixed-Earth reference frame and also applies corrections for annual aberration, diurnal aberration, light travel time, parallactic refraction and satellite phase.

Time instants t_i as obtained from programme "time reduction" are converted into MJD, taking the observation date into account. This yields $(MJD/\text{station})_i$.

The correction for light travel time is applied to form $(MJD/\text{satellite})_i$:

$$(MJD/\text{satellite})_i = (MJD/\text{station})_i - \frac{r_i}{2590 \times 10^7} \quad \ldots\ldots \quad (5.1)$$

where r_i stands for the estimated station-to-satellite range at t_i in km.

$(MJD/\text{satellite})_i$ will be abbreviated to MJD. For these MJD the Besselian Day Numbers C en D are linearly interpolated from the Astronomical Ephemeris [4].

Annual aberration is corrected for by adding corrections $\Delta_1\alpha$, $\Delta_1\delta$ to the α, δ – output of programme "position reduction" (section 4):

$$\left.\begin{aligned} \alpha' &= \alpha+\Delta_1\alpha \\ \delta' &= \delta+\Delta_1\delta \end{aligned}\right\} \qquad (5.2)$$

in which:

$$\left.\begin{aligned} \Delta_1\alpha &= Cc + Dd \\ \Delta_1\delta &= Cc' + Dd' \end{aligned}\right\} \qquad (5.3)$$

with:

$$\begin{aligned} c &= \cos\alpha\sec\delta \\ d &= \sin\alpha\sec\delta \\ c' &= \tan\varepsilon\cos\delta - \sin\alpha\sin\delta \\ d' &= \cos\alpha\sin\delta \end{aligned}$$

and $\varepsilon = 23^\circ.4425$ is the obliquity of the ecliptic.

In unit-vector form:

$$\underline{z} = \begin{pmatrix} \cos\delta'\cos\alpha' \\ \cos\delta'\sin\alpha' \\ \sin\delta' \end{pmatrix} \qquad (5.4)$$

MJD, which was calculated in terms of *UTC* is reduced to *MJD*1 in terms of *UT*1, by application of differences *UT*1–*UTC* obtained from a linear interpolation in the smoothed values as listed in circular D issued by the BIH.

Next define:

$$T = MJD1 - 33282 \qquad (5.5)$$

Precession is taken into account by matrix:

$$P = \begin{pmatrix} -\sin\varkappa\sin\omega+\cos\varkappa\cos\omega\cos\nu & -\cos\varkappa\sin\omega-\sin\varkappa\cos\omega\cos\nu & -\cos\omega\sin\nu \\ +\sin\varkappa\cos\omega+\cos\varkappa\sin\omega\cos\nu & +\cos\varkappa\cos\omega-\sin\varkappa\sin\omega\cos\nu & -\sin\omega\sin\nu \\ \cos\varkappa\sin\nu & -\sin\varkappa\sin\nu & +\cos\nu \end{pmatrix} \qquad (5.6)$$

in which:

$$\begin{aligned} \kappa &= 0''.063107T \\ \omega &= 0''.063107T \\ \nu &= 0''.054875T \end{aligned}$$

Nutation is accounted for by:

$$N = \begin{pmatrix} 1 & -\Delta\mu & -\Delta\nu \\ \Delta\mu & 1 & -\Delta\varepsilon \\ \Delta\nu & \Delta\varepsilon & 1 \end{pmatrix} \qquad (5.7)$$

with:

$$\Delta\mu = -76.7\times10^{-6}\sin\Psi_1 + 0.9\times10^{-6}\sin 2\Psi_1 - 5.7\times10^{-6}\sin 2\Psi_2 - 0.9\times10^{-6}\sin 2\Psi_3$$

$$\Delta\nu = -33.3\times10^{-6}\sin\Psi_1 + 0.4\times10^{-6}\sin 2\Psi_1 - 2.5\times10^{-6}\sin 2\Psi_2 - 0.4\times10^{-6}\sin 2\Psi_3$$

$$\Delta\varepsilon = +44.7\times10^{-6}\cos\Psi_1 - 0.4\times10^{-6}\cos 2\Psi_1 + 2.7\times10^{-6}\cos 2\Psi_2 + 0.4\times10^{-6}\cos 2\Psi_3$$

where:

$$\begin{aligned}\Psi_1 &= 12^\circ.1128 - 0^\circ.052954T\\ \Psi_2 &= 280^\circ.0812 + 0^\circ.985647T\\ \Psi_3 &= 64^\circ.3824 + 13^\circ.176396T\end{aligned}$$

Earth-rotation is expressed by

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix} \qquad (5.8)$$

with:

$$\begin{aligned}\theta = {} & 100^\circ.075542 +\\ & +360^\circ.9856473348T\\ & + 0^\circ.2900\times10^{-12}T^2\\ & - 4^\circ.392\times10^{-3}\sin\Psi_1\\ & + 0^\circ.053\times10^{-3}\sin 2\Psi_1\\ & - 0^\circ.325\times10^{-3}\sin 2\Psi_2\\ & - 0^\circ.050\times10^{-3}\sin 2\Psi_3\end{aligned}$$

Polar motion components x, y are taken from the smoothed values listed in circular D issued by BIH by means of a linear interpolation.

Polar motion matrix:

$$S = \begin{pmatrix} 1 & 0 & +x\\ 0 & 1 & -y\\ -x & +y & 1\end{pmatrix} \qquad (5.9)$$

The resultant rotation due to precession, nutation, earth rotation and polar motion is applied to unit vector $\underline{z}$ to give fixed-Earth direction $\underline{x}$:

$$\underline{x} = S\cdot R\cdot N\cdot P\cdot \underline{z} \qquad (5.10)$$

The procedure contained in formulas (5.5) through (5.10) follows [5].

Solve $\bar{\alpha}$, $\bar{\delta}$ from:

$$\begin{pmatrix} \cos\bar{\delta}\cos\bar{\alpha}\\ \cos\bar{\delta}\sin\bar{\alpha}\\ \sin\bar{\delta}\end{pmatrix} = \underline{x} \qquad (5.11)$$

where $\bar{\alpha}$ and $\bar{\delta}$ are the direction components of the directions to the satellite in the fixed-Earth (Greenwich) system.

If λ stands for the east-longitude of the station, then a sufficient approximation to the hour angle of the satellite is:

$$h = \lambda - \bar{\alpha} \qquad (5.12)$$

Corrections for diurnal aberration are [4]:

$$\left.\begin{aligned} \Delta_2\alpha &= 0''.32 \cos\varphi \cos h \sec\bar{\delta} \\ \Delta_2\delta &= 0''.32 \cos\varphi \sin h \sin\bar{\delta} \end{aligned}\right\} \qquad (5.13)$$

where φ is the latitude of the station.

The correction for parallactic refraction is calculated as follows (see [6]):

$$\cos z = \sin\varphi \sin\bar{\delta} + \cos\varphi \cos\bar{\delta} \cos h$$

$$\sin z = \sqrt{1-\cos^2 z}$$

$$\sin q = \frac{\sin h \cos\varphi}{\sin z}$$

$$\cos q = \frac{\sin\varphi - \sin\bar{\delta}\cos z}{\cos\bar{\delta}\sin z}$$

$$\Delta R = -435'' \cdot \frac{\sin z}{\cos^2 z} \cdot \frac{1}{r} \qquad (5.14)$$

$$\left.\begin{aligned} \Delta_3\alpha &= -\Delta R \cdot \sec\bar{\delta} \sin q \\ \Delta_3\delta &= -\Delta R \cdot \cos q \end{aligned}\right\} \qquad (5.15)$$

Incidentally, z stands for the zenith-angle of the station-to-satellite direction.

Satellite phase is corrected for as follows:

$$\left.\begin{aligned} \Delta_4\alpha &= 146'' \frac{\sin(\bar{\alpha}+h_\odot)\cos\delta_\odot}{w\cos\bar{\delta}} \cdot \frac{\varrho}{r} \\ \Delta_4\delta &= \frac{146'' \sin\bar{\delta}\cos\delta_\odot \cos(\bar{\alpha}+h_\odot) - \cos\bar{\delta}\sin\delta_\odot}{w} \cdot \frac{\varrho}{r} \end{aligned}\right\} \qquad (5.16)$$

with

$$w = \sqrt{1-\cos\bar{\delta}\cos\delta_\odot \cos(\bar{\alpha}+h_\odot) - \sin\bar{\delta}\sin\delta_\odot}$$

Here ϱ is the radius of the satellite in metres and $h_\odot$ and $\delta_\odot$ are the Greenwich ($\lambda = 0$) hour angle and the declination of the sun respectively.

$h_\odot$ is obtained from

$$h_\odot = \theta - \alpha_\odot \qquad (5.17)$$

and $\alpha_\odot$ and $\delta_\odot$ as solution of

$$\begin{pmatrix}\cos\delta_\odot\cos\alpha_\odot\\ \cos\delta_\odot\sin\alpha_\odot\\ \sin\delta_\odot\end{pmatrix}=\begin{pmatrix}\cos\lambda_\odot\\ \cos\varepsilon\sin\lambda_\odot\\ \sin\varepsilon\sin\lambda_\odot\end{pmatrix} \quad (5.18)$$

where $\varepsilon = 23°.4425$ is the obliquity of the ecliptic and $\lambda_\odot$ is the sun's longitude.

Finally:

$$\left.\begin{aligned}[\alpha] &= \bar{\alpha}+\Delta_2\alpha+\Delta_3\alpha+\Delta_4\alpha\\ [\delta] &= \bar{\delta}+\Delta_2\delta+\Delta_3\delta+\Delta_4\delta\end{aligned}\right\} \quad (5.19)$$

is the main result of programme "Conversion to fixed-Earth reference".

In unit-vector form:

$$\underline{l}=\begin{pmatrix}\cos[\delta]\cos[\alpha]\\ \cos[\delta]\sin[\alpha]\\ \sin[\delta]\end{pmatrix} \quad (5.20)$$

The variance-covariance matrix of $\underline{l}$ is estimated as:

$$V\{\underline{l}\} = G\cdot V\begin{Bmatrix}\alpha\\ \delta\end{Bmatrix}\cdot G^* \quad (5.21)$$

in which:

$$G=\begin{pmatrix}-\cos[\delta]\sin[\alpha] & -\sin[\delta]\cos[\alpha]\\ +\cos[\delta]\cos[\alpha] & -\sin[\delta]\sin[\alpha]\\ 0 & \cos[\delta]\end{pmatrix} \quad (5.22)$$

and where

$$V\begin{Bmatrix}\alpha\\ \delta\end{Bmatrix}$$

is taken from the output of programme "position reduction" (see section 4).

6 Quality assessment by means of curve-fitting

The combined "observations" t_i from (3.5) and α_i, δ_i from (4.14) are being checked on their internal precision by means of a curve fitting procedure.

Define:

$$\underline{v}_i=\begin{pmatrix}\cos\delta_i\cos\alpha_i\\ \cos\delta_i\sin\alpha_i\\ \sin\delta_i\end{pmatrix} \quad (6.1)$$

The direction cosines from (6.1) are referenced to a right-handed rectangular Cartesian frame which is defined by the first and the last directions observed on one plate, as follows:

$$\left.\begin{aligned} \underline{x} &\equiv \underline{v}_1 \\ \underline{z} &= \frac{\underline{v}_1 \times \underline{v}_n}{|\underline{v}_1 \times \underline{v}_n|} \\ \underline{y} &= \underline{z} \times \underline{x} \end{aligned}\right\} \qquad (6.2)$$

$$\underline{v}_i' = \begin{pmatrix} \underline{x}^* \\ \underline{y}^* \\ \underline{z}^* \end{pmatrix} \cdot \underline{v}_i \qquad (6.3)$$

Determine spherical coordinates along track and across track ξ_i and η_i from:

$$\begin{pmatrix} \cos\eta_i \cos\xi_i \\ \cos\eta_i \sin\xi_i \\ \sin\eta_i \end{pmatrix} = \underline{v}_i' \qquad (6.4)$$

$$0 \leqslant \xi_i < 360^\circ$$
$$-90^\circ \leqslant \eta_i \leqslant +90^\circ$$

Now, to ξ_i, η_i together with the t_i a curve fitting procedure is applied, as follows:

$$\tau_i = t_i - t_1 \qquad (6.5)$$

Define:

$$T_k = \begin{pmatrix} 1 & \tau_1 & \tau_1^2 & \tau_1^3 \ldots \tau_1^k \\ 1 & \tau_2 & \tau_2^2 & \tau_2^3 \ldots \tau_2^k \\ 1 & \tau_3 & \tau_3^2 & \tau_3^3 \ldots \tau_3^k \\ \vdots & & & \\ 1 & \tau_i & \tau_i^2 & \tau_i^3 \ldots \tau_i^k \\ \vdots & & & \\ 1 & \tau_n & \tau_n^2 & \tau_n^3 \ldots \tau_n^k \end{pmatrix} \qquad (6.6)$$

Then, assuming correlation freedom and unit weight within both observation vectors:

$$\underline{\xi} = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_n \end{pmatrix}; \qquad \underline{\eta} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{pmatrix} \qquad (6.7)$$

least squares solutions for the coefficient-vectors $\underline{a}$ and $\underline{b}$ are obtained from:

$$\begin{aligned} \underline{a} &= Q_k \cdot T_k^* \cdot \underline{\xi} \\ \underline{b} &= Q_k \cdot T_k^* \cdot \underline{\eta} \end{aligned} \qquad (6.8)$$

in which:

$$Q_k = (T_k^* \cdot T_k)^{-1} \qquad (6.9)$$

The correction vectors are:

$$\left.\begin{aligned} \underline{\varepsilon}_\xi &= T_k \cdot \underline{a} - \underline{\xi} \\ \underline{\varepsilon}_\eta &= T_k \cdot \underline{b} - \underline{\eta} \end{aligned}\right\} \quad \ldots \quad (6.10)$$

Here it has been tacitly assumed that the t_i are non-stochastic quantities.

Finally

$$\left.\begin{aligned} \hat{\sigma}_\xi^2 &= \frac{\underline{\varepsilon}_\xi^* \cdot \underline{\varepsilon}_\xi}{n-k-1} \\ \hat{\sigma}_\eta^2 &= \frac{\underline{\varepsilon}_\eta^* \cdot \underline{\varepsilon}_\eta}{n-k-1} \end{aligned}\right\} \quad \ldots \quad (6.11)$$

where n stands for the number of reduced satellite images and k for the degree of polynomial applied.

The estimates $\hat{\sigma}_\xi$ and $\hat{\sigma}_\eta$ are used for judging the "quality" of the photographic observations on each individual plate.

Noticing the small camerafield $k = 2$ was adopted invariably.

7 Results

All plates contributed to WEST, NGSP and ISAGEX have been listed in tables I (station WIPPOLDER) and II (station YPENBURG). It should be noted that listed observations of passive satellites Echo-1, Echo-2 and Pageos are essentially simultaneous with at least one other station. In particular do these tables give $\hat{\sigma}_\xi$ and $\hat{\sigma}_\eta$ for each individual plate, together with the number n of individual satellite images from which $\hat{\sigma}_\xi$ and $\hat{\sigma}_\eta$ have been calculated.

With the re-location by 1st December 1969 a new and conceptually better equatorial mount was put into use. Moreover an improved time-recording system was used in conjunction with the majority of observations made at YPENBURG.

Therefore it seems justified to make a break-down of the $\hat{\sigma}_\xi$- and $\hat{\sigma}_\eta$-values according the observation site (WIPPOLDER or YPENBURG). Moreover it makes sense to distinguish observations of passive from those of flashing satellites. Thus eight empirical relative frequency distributions are obtained (figs. 3, 4, 5, 6, 7, 8, 9 and 10).

$\hat{\sigma}_\xi$ and $\hat{\sigma}_\eta$ are taken as indicators of gross-errors either in the observations or in their reduction. Experience has suggested that observations with either $\hat{\sigma}_\xi$ and $\hat{\sigma}_\eta > 1''$ should be suspected. Adopting this rather arbitrary criterion it is meaningful to consider the percentage of observations (plates) with $\hat{\sigma}_\xi$ and/or $\hat{\sigma}_\eta < 1''$. The following conclusions are then to be drawn.

The fraction of observations with $\hat{\sigma}_\eta < 1''$ exceeds that with $\hat{\sigma}_\xi < 1''$. This is a well-known feature as far as passive satellites are concerned. It is less obvious for flashing satellites, where relative timing errors should not play a significant role.

For the YPENBURG-station the fraction of unsuspected observations ($\hat{\sigma}_\xi$ and/or $\hat{\sigma}_\eta < 1''$) exceeds that for the WIPPOLDER-station. This was to be expected when considering the improvements introduced by the re-location.

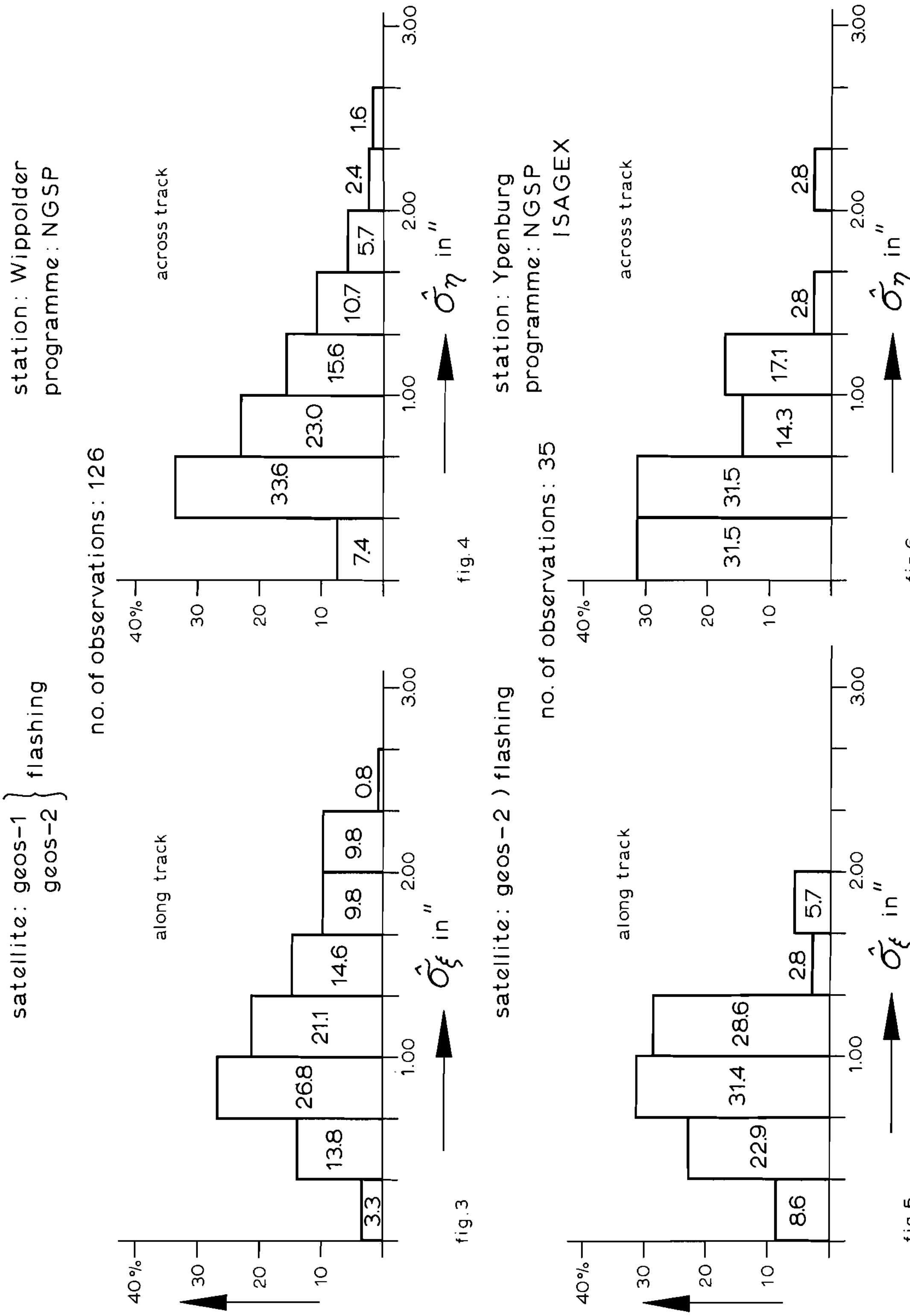
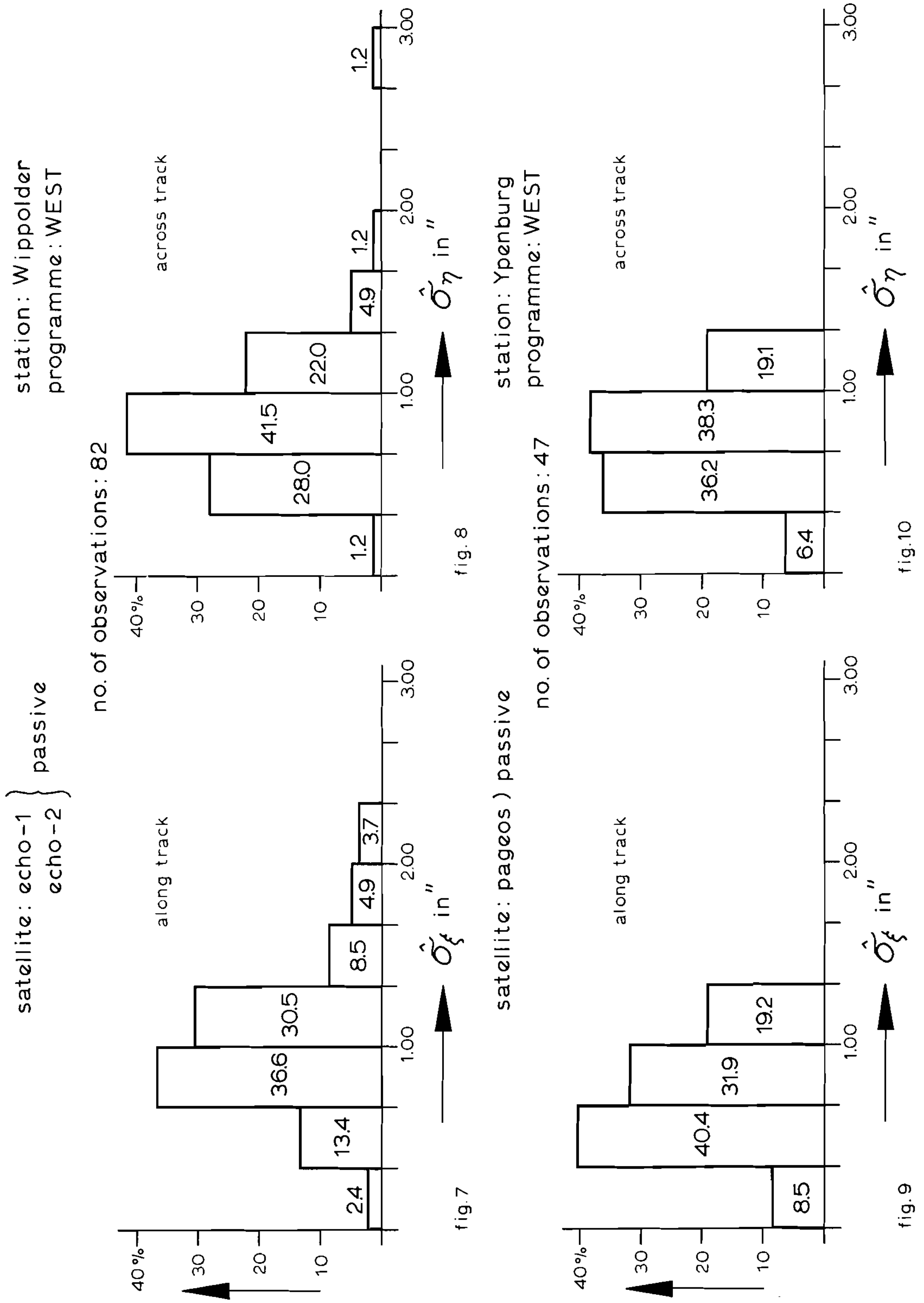


fig.3 fig.4 fig.5 fig.6


satellite: echo-1 / echo-2 } passive
station: Wippolder
programme: WEST
no. of observations: 82
along track
across track
40%
30
20
10
2.4
13.4
36.6
30.5
8.5
4.9
3.7
1.00
2.00
3.00
$\hat{\sigma}_\xi$ in"
1.2
28.0
41.5
22.0
4.9
1.2
1.2
$\hat{\sigma}_\eta$ in"
fig.7
fig.8
satellite: pageos) passive
station: Ypenburg
programme: WEST
no. of observations: 47
8.5
40.4
31.9
19.2
6.4
36.2
38.3
19.1
fig.9
fig.10

References

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[5] G. Veis – Precise aspects of terrestrial and celestial reference frames. In: The use of artificial satellites for geodesy. (ed. G. Veis). North-Holland Publishing Co., Amsterdam, 1963, pp. 201–216.

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Table I. Wippolder

plate no.	observation date	time	programme	satellite-name	$\hat{\sigma}_\xi$ in ″	$\hat{\sigma}_\eta$ in ″	n
24	660119	21h14m	NGSP	Geos-1	2.9	1.6	4
26	660425	22 55	NGSP	Geos-1	0.6	0.6	6
49	660716	23 46	NGSP	Geos-1	0.5	0.7	4
50	660716	23 51	NGSP	Geos-1	0.4	0.9	6
51	660725	22 15	NGSP	Geos-1	1.6	0.7	4
52	660725	22 21	NGSP	Geos-1	1.1	1.1	7
53	660727	22 28	NGSP	Geos-1	0.7	0.5	7
54	660728	00 30	NGSP	Geos-1	0.9	0.6	6
57	660903	20 59	NGSP	Geos-1	5.2	3.0	6
58	660903	21 06	NGSP	Geos-1	1.5	2.2	7
59	660916	21 07	WEST	Pageos	1.1	0.6	27
60	660916	21 37	WEST	Echo-2	0.7	1.0	5
64	660918	21 21	WEST	Pageos	1.6	1.4	35
65	660918	22 04	WEST	Echo-2	0.7	0.5	10
67	660920	20 43	WEST	Echo-2	1.1	0.6	15
68	660920	21 44	WEST	Pageos	1.0	0.8	31
69	660920	22 31	WEST	Echo-2	0.9	0.7	11
70	660922	21 09	WEST	Echo-2	0.8	0.4	8
73	661010	19 43	WEST	Echo-2	0.6	0.9	10
75	661011	19 00	WEST	Echo-2	0.7	0.5	11
84	670211	21 36	WEST	Pageos	1.2	0.8	14
89	670322	19 44	WEST	Pageos	1.7	0.6	26
90	670330	20 36	WEST	Pageos	0.9	1.4	22
92	670401	20 47	WEST	Pageos	0.8	0.9	28
93	670401	21 24	WEST	Echo-2	0.7	0.8	10
95	670413	20 17	WEST	Echo-2	1.1	0.5	5
96	670413	22 07	WEST	Echo-1	1.0	0.8	9
98	670415	22 04	WEST	Echo-1	0.9	0.6	13
100	670423	21 31	WEST	Echo-1	1.1	1.1	13
101	670424	21 28	WEST	Echo-1	1.5	0.6	10
102	670425	20 45	WEST	Echo-2	1.8	1.8	18
103	670425	21 20	WEST	Echo-1	1.7	1.1	14
105	670612	23 44	WEST	Echo-2	1.4	0.9	17
106	670613	22 45	WEST	Echo-2	0.8	0.5	10
107	670616	00 35	WEST	Echo-2	0.8	0.4	6
110	670706	21 35	WEST	Echo-2	1.3	1.1	15
111	670707	22 50	WEST	Echo-1	1.3	1.0	20
112	670709	22 22	WEST	Echo-2	0.5	1.2	16
113	670710	21 28	WEST	Echo-2	0.4	0.9	14
114	670712	21 24	WEST	Echo-2	0.8	0.9	9
116	670820	23 20	WEST	Pageos	0.6	0.9	17
117	670828	00 30	WEST	Echo-1	1.7	1.0	13
118	670829	02 12	WEST	Echo-1	1.4	0.7	10
119	670830	00 08	WEST	Echo-1	1.3	0.8	15
120	670906	20 49	WEST	Pageos	1.3	1.4	13
121	670909	01 32	WEST	Echo-2	1.0	0.5	11
122	670910	19 39	WEST	Echo-1	0.6	0.8	15
123	670910	20 58	WEST	Pageos	0.6	0.5	13
124	670917	19 55	WEST	Echo-1	1.0	0.5	8
129	670924	21 25	WEST	Pageos	1.0	1.0	12
131	670927	02 02	WEST	Echo-2	0.8	0.7	17
132	670927	19 24	WEST	Echo-1	2.0	0.7	15
133	670927	21 14	WEST	Echo-1	0.7	0.7	10
134	670928	20 58	WEST	Echo-1	1.0	1.3	13

plate no.	observation date	time	programme	satellite-name	$\hat{\sigma}_\xi$ in ″	$\hat{\sigma}_\eta$ in ″	n
135	670929	00h05m	WEST	Echo-2	1.2	0.8	9
136	671001	20 10	WEST	Echo-1	1.6	2.9	9
138	671013	18 53	WEST	Pageos	1.1	1.0	14
144	671108	18 34	WEST	Echo-2	1.5	0.7	12
147	671109	03 26	WEST	Echo-1	1.2	0.7	15
153	671113	02 09	WEST	Pageos	1.1	1.1	6
155	671117	18 17	WEST	Echo-2	1.3	0.7	11
157	671118	03 25	WEST	Echo-2	0.7	1.0	8
160	671120	18 40	WEST	Echo-2	1.1	1.1	11
161	671121	02 19	WEST	Pageos	0.9	1.6	5
165	671220	00 04	WEST	Pageos	2.3	0.4	7
166	671220	17 18	WEST	Echo-1	0.5	0.7	11
170	680208	18 08	WEST	Pageos	0.7	0.7	6
172	680218	17 21	WEST	Echo-1	0.8	0.9	8
173	680220	19 32	NGSP	Geos-2	0.8	1.5	5
174	680225	19 21	NGSP	Geos-2	0.9	0.3	4
175	680226	19 37	NGSP	Geos-2	1.6	1.6	7
176	680226	19 41	NGSP	Geos-2	1.5	0.4	4
177	680303	19 45	NGSP	Geos-2	1.2	1.3	4
178	680304	20 01	NGSP	Geos-2	1.9	1.5	6
179	680310	20 06	NGSP	Geos-2	2.0	0.6	7
180	680310	20 10	NGSP	Geos-2	0.8	0.2	4
182	680320	19 42	NGSP	Geos-2	1.2	0.5	4
183	680321	20 00	NGSP	Geos-2	1.6	0.5	4
184	680322	20 16	NGSP	Geos-2	2.0	3.0	5
185	680322	20 19	NGSP	Geos-2	0.9	0.7	4
186	680324	20 56	NGSP	Geos-2	0.8	1.8	6
187	680326	19 45	NGSP	Geos-2	0.2	0.5	4
188	680326	19 49	NGSP	Geos-2	2.0	1.0	6
189	680327	02 17	WEST	Echo-2	0.8	1.0	8
191	680327	20 10	NGSP	Geos-2	1.4	0.9	7
192	680327	20 05	NGSP	Geos-2	1.9	1.3	4
193	680328	01 03	WEST	Echo-2	1.0	0.6	11
194	680328	02 49	WEST	Echo-2	0.9	0.6	15
195	680328	20 24	NGSP	Geos-2	1.8	0.5	4
196	680329	01 35	WEST	Echo-2	0.8	0.9	11
197	680405	21 06	NGSP	Geos-2	0.9	0.5	6
198	680406	21 25	NGSP	Geos-2	0.2	2.0	5
199	680406	21 28	NGSP	Geos-2	1.5	0.5	5
200	680407	21 45	NGSP	Geos-2	1.0	1.5	6
201	680408	20 11	NGSP	Geos-2	2.0	1.4	7
203	680408	20 17	NGSP	Geos-2	1.4	0.5	6
204	680409	20 34	NGSP	Geos-2	1.6	1.8	4
205	680409	20 38	NGSP	Geos-2	1.7	0.4	7
206	680412	21 31	NGSP	Geos-2	2.0	0.8	6
210	680413	21 49	NGSP	Geos-2	0.7	2.5	6
211	680415	20 39	NGSP	Geos-2	2.0	0.6	4
212	680415	20 43	NGSP	Geos-2	2.2	1.1	7
213	680420	21 31	WEST	Echo-1	1.3	1.3	7
214	680421	00 09	WEST	Echo-2	0.7	1.3	10
215	680424	21 39	NGSP	Geos-2	0.5	0.9	6
216	680424	21 43	NGSP	Geos-2	0.8	0.3	5
217	680425	22 02	NGSP	Geos-2	0.8	0.7	6
219	680427	00 30	NGSP	Geos-2	1.6	0.9	6
221	680505	21 29	NGSP	Geos-2	1.0	0.9	7

plate no.	observation date	time	programme	satellite-name	$\hat{\sigma}_\xi$ in ″	$\hat{\sigma}_\eta$ in ″	n
222	680505	21h31m	NGSP	Geos-2	0.8	0.4	4
223	680505	21 33	NGSP	Geos-2	0.8	0.5	5
225	680512	21 53	NGSP	Geos-2	1.6	1.4	7
226	680512	21 55	NGSP	Geos-2	2.0	1.8	5
227	680512	21 59	NGSP	Geos-2	2.0	0.4	6
229	680519	22 18	NGSP	Geos-2	0.6	0.5	7
230	680519	22 22	NGSP	Geos-2	1.2	0.4	5
231	680520	22 41	NGSP	Geos-2	2.3	0.4	5
232	680521	00 14	WEST	Pageos	0.8	0.5	9
233	680525	00 07	WEST	Pageos	0.6	1.0	7
234	680602	00 11	WEST	Pageos	0.5	0.9	10
235	680605	00 09	WEST	Pageos	0.8	0.8	12
236	680612	23 56	WEST	Pageos	0.7	0.8	9
237	680615	01 43	WEST	Echo-2	0.7	0.7	8
238	680622	23 53	WEST	Pageos	1.0	0.5	4
241	680704	23 45	WEST	Echo-2	0.3	0.2	5
242	680709	21 50	WEST	Echo-2	1.0	0.5	6
243	680717	00 26	NGSP	Geos-2	1.2	0.7	6
244	680723	23 02	NGSP	Geos-2	0.6	0.4	7
245	680728	22 49	NGSP	Geos-2	0.8	0.4	6
246	680809	22 58	NGSP	Geos-2	1.0	0.5	6
247	680810	23 18	NGSP	Geos-2	0.5	0.5	6
248	680822	23 28	NGSP	Geos-2	1.0	1.0	5
249	681007	23 29	NGSP	Geos-2	0.9	0.5	5
250	681014	23 53	NGSP	Geos-2	0.5	0.9	6
252	681021	23 16	WEST	Pageos	0.9	0.4	7
253	681022	00 16	NGSP	Geos-2	1.9	1.9	6
254	681022	02 06	NGSP	Geos-2	0.9	0.3	4
255	681105	18 12	NGSP	Geos-2	1.0	0.6	6
257	681109	17 40	NGSP	Geos-2	0.8	1.0	5
258	681110	00 49	NGSP	Geos-2	1.2	0.9	6
259	681110	02 40	NGSP	Geos-2	3.2	1.0	4
260	681113	01 47	NGSP	Geos-2	0.7	0.6	4
261	681113	03 38	NGSP	Geos-2	1.8	0.5	5
262	681113	18 54	NGSP	Geos-2	1.0	0.4	6
263	681114	00 18	NGSP	Geos-2	2.0	0.7	6
264	681114	02 09	NGSP	Geos-2	2.0	1.0	7
265	681115	17 44	NGSP	Geos-2	0.6	1.0	4
266	681121	17 45	NGSP	Geos-2	1.8	1.2	6
267	681121	17 49	NGSP	Geos-2	1.4	0.4	5
270	681122	01 00	NGSP	Geos-2	1.1	0.7	6
271	681219	19 24	NGSP	Geos-2	1.0	0.9	7
272	681219	19 29	NGSP	Geos-2	0.7	1.0	7
273	681221	20 05	NGSP	Geos-2	1.0	0.9	6
274	690103	01 36	NGSP	Geos-2	0.9	1.0	7
275	690103	03 28	NGSP	Geos-2	1.2	0.8	7
276	690107	02 54	NGSP	Geos-2	1.3	0.6	7
277	690109	03 33	NGSP	Geos-2	1.6	0.9	7
278	690114	18 35	NGSP	Geos-2	0.7	0.9	6
279	690131	20 21	NGSP	Geos-2	1.7	1.2	5
280	690223	20 20	NGSP	Geos-2	1.4	0.8	7
281	690223	20 25	NGSP	Geos-2	1.4	0.6	5
282	690304	19 33	NGSP	Geos-2	1.9	1.2	7
283	690304	21 25	NGSP	Geos-2	1.9	1.4	6
284	690306	20 13	NGSP	Geos-2	0.5	0.6	5

plate no.	observation date	time	programme	satellite-name	$\hat{\sigma}_\xi$ in ″	$\hat{\sigma}_\eta$ in ″	n
285	690311	19h57m	NGSP	Geos-2	1.2	0.7	7
286	690311	21 48	NGSP	Geos-2	1.9	1.4	6
287	690322	19 49	NGSP	Geos-2	0.9	1.8	7
288	690325	20 47	NGSP	Geos-2	1.5	1.3	6
289	690325	20 51	NGSP	Geos-2	0.2	1.0	5
291	690403	20 01	NGSP	Geos-2	0.4	0.8	5
292	690403	23 25	WEST	Pageos	0.7	0.7	7
293	690404	20 19	NGSP	Geos-2	1.0	2.3	5
294	690404	20 23	NGSP	Geos-2	0.8	0.7	5
296	690405	20 44	NGSP	Geos-2	0.9	1.8	5
297	690405	22 27	NGSP	Geos-2	0.7	1.4	7
298	690405	22 31	NGSP	Geos-2	1.0	2.4	5
299	690407	21 14	NGSP	Geos-2	2.0	0.5	7
300	690407	21 19	NGSP	Geos-2	0.7	0.2	4
301	690408	21 37	NGSP	Geos-2	0.5	0.1	4
302	690409	21 53	NGSP	Geos-2	0.9	1.4	6
303	690409	21 58	NGSP	Geos-2	1.3	1.6	4
306	690416	22 22	NGSP	Geos-2	0.4	0.4	4
307	690416	23 12	WEST	Pageos	0.3	1.0	7
308	690417	20 52	NGSP	Geos-2	0.6	0.6	6
309	690417	22 38	NGSP	Geos-2	0.3	1.2	5
310	690417	23 10	WEST	Pageos	2.2	0.8	5
311	690427	22 11	NGSP	Geos-2	0.7	0.3	4
313	690428	22 59	WEST	Echo-2	0.7	0.9	8
314	690428	23 04	WEST	Pageos	0.8	0.8	9
315	690429	21 01	NGSP	Geos-2	1.2	0.5	5
317	690430	21 17	NGSP	Geos-2	0.8	0.7	4
318	690430	21 21	NGSP	Geos-2	0.8	1.4	5
319	690503	22 13	NGSP	Geos-2	1.2	0.2	4
321	690507	21 41	NGSP	Geos-2	0.5	0.8	4
322	690508	21 58	NGSP	Geos-2	1.1	0.6	6
323	690508	22 00	NGSP	Geos-2	1.4	0.2	4
324	690508	22 06	NGSP	Geos-2	0.9	1.3	5
325	690504	22 24	NGSP	Geos-2	1.9	1.4	5
326	690513	21 45	NGSP	Geos-2	1.3	0.9	5
327	690523	23 08	NGSP	Geos-2	0.7	0.4	4
328	690527	00 04	NGSP	Geos-2	0.5	0.8	6
329	690605	23 37	NGSP	Geos-2	1.1	0.4	4
330	690714	23 21	NGSP	Geos-2	2.3	1.3	5
331	690715	23 39	NGSP	Geos-2	1.5	1.7	4
332	690718	22 43	NGSP	Geos-2	0.4	0.5	5
333	690808	00 38	WEST	Pageos	1.6	0.6	5
334	690810	00 38	WEST	Pageos	0.4	0.7	9
335	690811	00 40	WEST	Pageos	0.6	0.4	9

Table II. Ypenburg

plate no.	observation date	time	programme	satellite-name	$\hat{\sigma}_\xi$ in ″	$\hat{\sigma}_\eta$ in ″	n
336	691217	04h48m	NGSP	Geos-2	1.3	0.3	4
337	691219	03 35	NGSP	Geos-2	0.7	0.3	5
338	691219	03 39	NGSP	Geos-2	1.1	0.3	6
339	691219	05 25	NGSP	Geos-2	0.4	1.3	6
340	700104	03 12	NGSP	Geos-2	1.0	0.4	6
341	700104	05 01	NGSP	Geos-2	0.8	0.6	6
342	700104	05 07	NGSP	Geos-2	1.3	1.1	7
343	700107	06 01	NGSP	Geos-2	1.4	0.8	5
345	700110	03 17	NGSP	Geos-2	0.4	0.2	4
347	700203	21 59	WEST	Pageos	0.7	1.1	8
348	700204	00 58	WEST	Pageos	0.8	0.8	9
350	700217	01 03	WEST	Pageos	0.1	1.3	5
352	700310	21 55	WEST	Pageos	0.5	0.5	8
353	700310	22 03	WEST	Pageos	1.0	0.9	12
355	700325	21 44	WEST	Pageos	0.7	0.7	10
356	700325	21 55	WEST	Pageos	1.0	0.5	11
359	700603	00 52	WEST	Pageos	1.2	0.9	8
360	700603	01 01	WEST	Pageos	0.5	0.7	7
361	700604	00 52	WEST	Pageos	0.4	0.4	7
362	700604	01 02	WEST	Pageos	0.8	0.6	10
363	700605	00 54	WEST	Pageos	1.2	0.2	6
364	700606	00 59	WEST	Pageos	0.4	1.0	10
365	700607	00 56	WEST	Pageos	0.6	0.9	9
366	700607	01 04	WEST	Pageos	0.6	1.1	11
367	700612	00 59	WEST	Pageos	1.0	0.6	6
368	700728	22 33	WEST	Pageos	0.8	0.6	6
369	700731	22 34	WEST	Pageos	0.5	0.6	6
370	700801	22 34	WEST	Pageos	0.5	0.6	9
371	700803	22 33	WEST	Pageos	0.6	0.8	10
373	700925	21 20	ISAGEX	Geos-2	0.9	0.3	5
374	700928	20 31	ISAGEX	Geos-2	0.5	2.0	4
381	701016	03 53	WEST	Pageos	1.3	0.4	9
385	701027	03 52	WEST	Pageos	0.1	0.7	5
388	700924	19 13	ISAGEX	Geos-2	1.0	1.0	4
390	710111	22 53	ISAGEX	Geos-2	0.3	0.4	4
391	710114	19 55	WEST	Pageos	0.6	0.8	11
392	710121	23 05	WEST	Pageos	0.5	1.2	7
394	710125	20 03	WEST	Pageos	0.9	0.7	7
398	710129	00 38	ISAGEX	Geos-2	0.4	0.5	5
400	710129	20 07	WEST	Pageos	0.8	0.7	7
411	710216	20 23	WEST	Pageos	0.6	0.4	10
414	710222	02 50	WEST	Pageos	0.7	1.0	9
418	710222	20 38	WEST	Pageos	0.7	0.6	9
421	710303	20 44	WEST	Pageos	0.7	1.0	7
422	710304	00 29	ISAGEX	Geos-2	0.6	0.4	6
423	710304	02 57	WEST	Pageos	1.0	0.7	9
431	710310	00 34	ISAGEX	Geos-2	0.5	1.1	7
432	710310	03 12	WEST	Pageos	1.1	1.3	8
437	710316	03 15	WEST	Pageos	0.9	0.8	8
440	710316	03 29	WEST	Pageos	0.6	0.8	7
441	710323	01 04	ISAGEX	Geos-2	0.7	1.4	5
444	710326	02 02	ISAGEX	Geos-2	0.5	0.3	4
447	710328	02 40	ISAGEX	Geos-2	2.9	0.2	4
450	710329	02 59	ISAGEX	Geos-2	0.9	0.1	4

plate no.	observation date	time	programme	satellite-name	$\hat{\sigma}_\xi$ in ″	$\hat{\sigma}_\eta$ in ″	n
453	710330	03h18m	ISAGEX	Geos-2	1.0	0.8	5
455	710412	01 57	ISAGEX	Geos-2	0.9	0.2	4
460	710415	00 48	WEST	Pageos	0.3	0.7	10
465	710415	02 52	ISAGEX	Geos-2	1.7	0.4	6
467	710419	02 19	ISAGEX	Geos-2	1.0	0.9	4
471	710421	00 58	WEST	Pageos	0.6	0.6	10
473	710421	01 09	ISAGEX	Geos-2	1.1	0.7	6
475	710421	02 57	ISAGEX	Geos-2	0.8	0.2	5
480	710422	00 57	WEST	Pageos	0.3	0.3	6
489	710428	03 21	ISAGEX	Geos-2	1.1	0.1	5
492	710504	01 16	WEST	Pageos	0.4	1.0	8
494	710506	02 16	ISAGEX	Geos-2	1.9	0.5	5
495	710510	22 19	WEST	Pageos	0.9	0.4	8
496	710511	22 31	WEST	Pageos	0.6	0.7	7
497	710513	22 33	WEST	Pageos	0.6	0.6	7
498	710521	22 46	WEST	Pageos	0.7	0.6	8
499	710522	01 45	WEST	Pageos	1.2	0.1	5
500	710528	23 10	WEST	Pageos	0.6	0.6	6
502	710602	23 26	WEST	Pageos	0.6	1.2	7
503	710603	23 30	WEST	Pageos	0.7	0.5	11
504	710607	23 35	WEST	Pageos	0.7	0.6	8
505	710725	21 09	ISAGEX	Geos-2	0.4	1.0	5
506	710726	21 30	ISAGEX	Geos-2	0.3	0.4	5
507	710731	21 17	ISAGEX	Geos-2	0.6	0.5	6
508	710802	21 56	ISAGEX	Geos-2	0.2	1.1	5
509	710808	21 57	ISAGEX	Geos-2	0.9	0.5	6
510	710816	20 54	ISAGEX	Geos-2	0.9	0.3	7
511	710824	21 37	ISAGEX	Geos-2	1.0	0.5	6
512	710825	20 05	ISAGEX	Geos-2	0.8	0.5	7