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# THE PRECISION OF PHOTOGRAMMETRIC MODELS

by

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CONTENTS

## SUMMARY

The influence of random observation errors on the machine coordinates of model points can be represented by a covariance matrix. In this investigation the covariance matrix of a limited number of model points has been determined in two different ways:

1. by executing repeated measurements of points in photogrammetric models; in some cases these repeated measurements are combined with repeated relative orientations or with repeated relative and inner orientations; from the series of these measurements the estimated covariance matrices ($\hat{\sigma}^2$) have been determined.
2. by writing the machine coordinates as functions of the initial observations, e.g. $x$- and $y$-parallaxes; by means of the standard deviations of these initial observations, and applying the law of propagation of errors, the covariance matrices ($\sigma^2$) have been computed.

1. Eight experiments have been executed to determine the ($\hat{\sigma}^2$). The following observations were repeated 20 times:

for 2 models: the measuring of the coordinates of 8 model points (after the inner and relative orientation was done once).

for 3 models: the relative orientation and the measuring of the coordinates of 8 model points (after inner orientation was done once).

for 3 models: the inner orientation, the relative orientation and the measuring of the coordinates of 8 model points.

The estimated covariance matrix ($\hat{\sigma}^2$) is computed from the 20 repeated observations per experiment. This makes 8 full-matrices of $24 \times 24$ elements for the $8 \times 3$ coordinates of the 8 model points.

2. The 8 covariance matrices ($\sigma^2$) were computed from the standard deviations by applying the law of propagation of errors. The observations are divided into three groups:

- the measuring of the coordinates of a model point
- the relative orientation
- the inner orientation

Sub-matrices of ($\hat{\sigma}^2$) and ($\sigma^2$) represent point standard ellipsoids and relative standard ellipsoids. The shape and position of these ellipsoids are represented and compared in a large number of diagrams showing the projections of the ellipsoids on three perpendicular planes. These projections are standard ellipses and relative standard ellipses.

Interesting correlations are demonstrated, both between coordinates of a single point and between coordinates of different points.

In order to be able to extrapolate these results further investigations will be necessary for better information about the factors which influence the standard deviations of the individual observations.

The structure of the covariance matrix of the coordinates of model points is essential for studies of precision and accuracy in all procedures which use the photogrammetric model as basic unit.

# I INTRODUCTION

Photogrammetric models, reconstructions of terrain models by means of stereophotographs, are often used as basic units for measurements. Data taken from a photogrammetric model are used in different procedures e.g.:
- strip- and block-triangulation
- determination of profiles
- digital terrain models

etc.

The object of this investigation is to study and analyse the influence of random observation errors on the coordinates of model points. The publication of R. ROELOFS, "Theory of errors of photogrammetric mapping, The Ohio State University, Columbus" [2] has been a stimulus and guide for the design and execution of this investigation.

The observations are partly stereoscopical and partly monocular. Needless to say that these errors are only part of the total group of errors which influence the accuracy of photogrammetric models. No attempt will be made to sum up all these errors.

The observation errors can be divided into three groups; errors due to:
1. the measuring of a model point
2. the relative orientation
2. the inner orientation

They can briefly be described as follows.

The first group: for measuring a model point, the measuring mark is set at the proper elevation and in the proper planimetric position.

The second group: for the relative orientation, the observer has to eliminate $y$-parallaxes in order to get an intersection of corresponding rays of the two bundles.

The third group: the inner orientation, is the positioning of the photo in the plate holder of the instrument and the setting of the proper principal distance to reconstruct the bundle of rays.

It is known that the observation errors depend on various characteristics: the instrument, the photographs, the observer, size and shape of model points, etc. Therefore this analysis will be restricted to:
- stereophotogrammetry with analogue instruments which have the possibility to produce data in the form of machine coordinates.
- only pricked points and signalized points are concerned in the investigation.
- the experiments are only made with a Wild A7 and a Wild A8.
- aerial photographs of "normal" quality and taken from nearly flat or hilly terrain are used.
- the measurements are done by two trained operators.

Special attention is paid to pricked points and signalized points because these points are often connection points in triangulations. In addition these points are symmetrical as distinct from many terrain points, such as corners of houses, intersections of roads, etc. The symmetry of points makes the observation data more or less homogeneous which simplifies the statistical description.

The precision of model points will be represented by the covariance matrix of the coordinates of model points. So the correlation between the coordinates of different points will also be implicated in the study. For the present only points of one model will be considered.

The covariance matrix of a limited number of points will be determined in two different ways:

1. from series of repeated measurements which can be considered as a probablility distribution of the same quantity: the coordinates of model points. The covariance matrix determined in this way will be called an estimate of the covariance matrix or the estimated covariance matrix ($\hat{\sigma}^2$).
2. by writing the machine coordinates as functions of the observations, the three groups of observations as described before, and applying the law of propagation of errors. This covariance matrix has no special adjective and will be indicated as ($\sigma^2$).

In this investigation these two covariance matrices will be studied and compared in order to come to a more general description of the precision of model points.

# II ERRORS DUE TO THE MEASURING OF A MODEL POINT

## 1 General description

For measuring the machine coordinates of a point in a model, the floating mark is set in the proper elevation and in the proper planimetric position. The error in elevation influences the error in planimetric position and reverse.

In appendix 1 the differential formula is derived which gives the relation between the differentials of the machine coordinates of a point, $\Delta x$, $\Delta y$ and $\Delta h$ and the differentials of the observations, which are:

the horizontal parallax: $\Delta p_x$
the $x$-setting: $\Delta x'$
the $y$-setting: $\Delta y'$

$$\begin{pmatrix} \Delta x_M \\ \Delta x_{ML} \\ \Delta x_{MR} \\ \Delta y_M \\ \Delta h_M \end{pmatrix} = \begin{pmatrix} -\frac{x}{b} & 1 & 0 \\ -\frac{1}{2b}(2x+b) & 1 & 0 \\ -\frac{1}{2b}(2x-b) & 1 & 0 \\ -\frac{y}{b} & 0 & 1 \\ +\frac{z}{b} & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta p_x \\ \Delta x' \\ \Delta y' \end{pmatrix} \quad \ldots\ldots\ldots\ldots \quad (2.1)$$

$x$ and $y$ are the coordinates of the model point; the origin of the coordinates is chosen in the middle, $O$, of the instrumental base $b$ of the instrument which coincides with the $x$-axes; see figure 2.1.

In this figure two systems of coordinates are introduced:

$x\,y\,z$: the origin of the coordinates is chosen in $O$;
$x\,y\,h$: the system of machine coordinates with – in principle – any position of the origin.

We distinguish three cases for the $x$-coordinate in formula (2.1):

$\Delta x_M$: signalized points.
$\Delta x_{ML}$ and $\Delta x_{MR}$: pricked points on left and right photograph respectively.

The differentials for the $y$- and $h$-coordinate are the same for signalized and pricked points. The index $M$ is introduced here for the differentials referring to measuring of a model point.

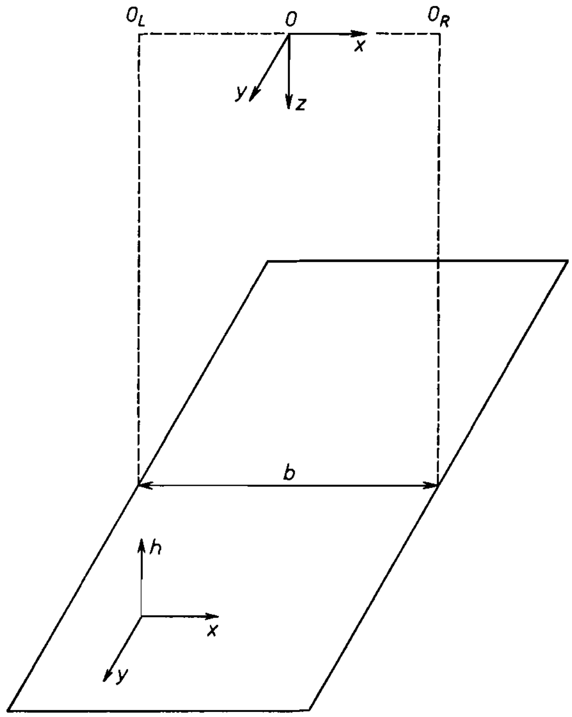


Fig. 2.1. Systems of coordinates in a photogrammetric model.

In (2.1) we replace:

$$\begin{pmatrix} \Delta x_M \\ \Delta x_{ML} \\ \Delta x_{MR} \\ \Delta y_M \\ \Delta h_M \end{pmatrix} \equiv (\Delta x_M^i) \quad \begin{pmatrix} -\frac{x}{b} & 1 & 0 \\ -\frac{1}{2b}(2x+b) & 1 & 0 \\ -\frac{1}{2b}(2x-b) & 1 & 0 \\ -\frac{y}{b} & 0 & 1 \\ +\frac{x}{b} & 0 & 0 \end{pmatrix} \equiv (A_M^i) \quad \begin{pmatrix} \Delta p_x \\ \Delta x' \\ \Delta y' \end{pmatrix} \equiv (\Delta M) \quad (2.2)$$

then:

$$(\Delta x_M^i) = (A_M^i)(\Delta M) \quad (2.3)$$

The covariance matrix of the machine coordinates can be computed by application of the law of propagation of errors to (2.3).

$$\left(\sigma_{x_M^i x_M^j}\right) = (A_M^i)(\sigma_M^2)(A_M^j)^T \quad (2.4)$$

$(\sigma_M^2)$ is the covariance matrix of the observations.

$\sigma_{\Delta p_x}$, $\sigma_{x'}$ and $\sigma_{y'}$ are the standard deviations of these observations respectively:
- the horizontal parallax: $\Delta p_x$
- the $x$-setting: $\Delta x'$
- the $y$-setting: $\Delta y'$

The observations are assumed to be free of correlation and for that the covariance matrix $(\sigma^2_M)$ is a diagonal matrix. The elements of the diagonal are made up by the square of the standard deviations.

The three standard deviations, expressed in microns in photo scale, are:

| | $\sigma_{\Delta p_x}$ | $\sigma_{x'}$ | $\sigma_{y'}$ |
|---|---|---|---|
| signalized points | 4.9 | 4.7 | 6.0 |
| pricked points | 6.5 | 4.2 | 6.5 |

. . . . . . . . . (2.5)

These values are determined from a large number of redundant observations obtained by repeated measurements in an orientated model.

The following two remarks have to be made here:
- In both cases the error in $y$-setting is larger than the error in $x$-setting, $\sigma_{y'} > \sigma_{x'}$; this may have a physiological cause.
- For the pricked points we see furthermore:

$\sigma_{\Delta p_x} > \sigma_{x'} \sqrt{2}$

This may be caused by the fact that the points are only pricked on one plate, which partly disturbs the stereoscopy.

Introduction of (2.5) into (2.4) makes it possible to compute the covariance matrix of the machine coordinates of all points concerned.

## 2 The experiments

In order to evaluate the covariance matrix $\left(\sigma_{x^i_M x^j_M}\right)$, as described in the previous section, two experiments I and II have been executed.

*Experiment I*
Stereopair: 1888–1887
Photo scale: 1:15000
Camera: Wild RC 5, $c = 152.47$ mm
Size: $23 \times 23$ cm
Distance model – projection centres: $z \simeq 250$ mm
Instrumental base: $b = 148$ mm

After an empirical relative orientation of this pair in a Wild A8, the machine coordinates of 8 pricked points were measured 20 times.

Figure 2.2 gives the position of the 8 pricked points on the two photographs. Points 2, 4, 6 and 8 are pricked on the left photo 1888 and points 1, 3, 5 and 7 on photo 1887. The position of the points in the model is given in figure 2.3.

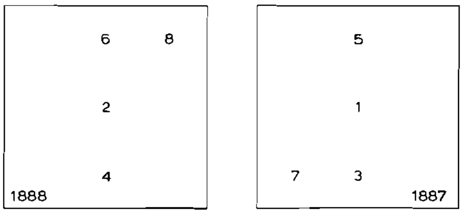


Fig. 2.2. The position of the 8 pricked points on the photographs.

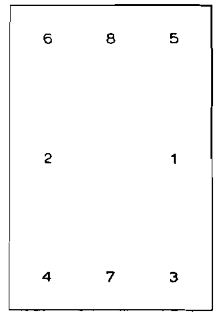


Fig. 2.3. The position of the points in the model.

The observations were made in a random sequence and automatically registered to the nearest 0.01 mm. The 20 observations of each point can be considered as a sample from the probability distribution of the coordinate-variates.

The observations are:

$$x_k^n, y_k^n \text{ and } h_k^n \quad \ldots\ldots\ldots\ldots \quad (2.6)$$

$n$: 1 to 8, number of points
$k$: 1 to 20, number of repetitions.

The mean values are:

$$x^n = \frac{1}{k}[x_k^n]_1^k \qquad y^n = \frac{1}{k}[y_k^n]_1^k \qquad h^n = \frac{1}{k}[h_k^n]_1^k \quad \ldots\ldots\ldots \quad (2.7)$$

From the differences with respect to the mean values, $x^n - x_k^n$, $y^n - y_k^n$ and $h^n - h_k^n$, an estimate for the covariance matrix can be computed:

$$\left(\hat{\sigma}_{x_M^i x_M^j}\right), \qquad i = 1, \ldots, 24 \quad \ldots\ldots\ldots \quad (2.8)$$

The elements of this matrix are (*page* 11):

Table 2.1. The elements of the estimated covariance matrix $\left(\hat{\sigma}_{x_M^i x_M^j}\right)$ of experiment I.

| | $\hat{\sigma}_{x_M^n x_M^n}$ | $\hat{\sigma}_{y_M^n y_M^n}$ | $\hat{\sigma}_{h_M^n h_M^n}$ | $\hat{\sigma}_{x_M^n y_M^n}$ | $\hat{\sigma}_{x_M^n h_M^n}$ | $\hat{\sigma}_{y_M^n h_M^n}$ |
|---|---|---|---|---|---|---|
| 1 | 54 | 98 | 487 | + 1 | − 48 | − 50 |
| 2 | 22 | 66 | 158 | − 5 | 0 | 0 |
| 3 | 66 | 283 | 226 | + 56 | − 44 | −221 |
| 4 | 51 | 188 | 174 | − 33 | + 18 | −171 |
| 5 | 131 | 150 | 620 | −106 | −232 | +259 |
| 6 | 62 | 73 | 352 | + 18 | + 9 | + 95 |
| 7 | 33 | 105 | 203 | + 10 | − 6 | − 84 |
| 8 | 31 | 136 | 213 | − 22 | − 38 | + 93 |

$$\left.\begin{aligned}
\hat{\sigma}_{x^n_M x^n_M} &= \frac{[(x^n - x^n_k)^2]^k_1}{k-1}\\
\hat{\sigma}_{y^n_M y^n_M} &= \frac{[(y^n - y^n_k)^2]^k_1}{k-1}\\
\hat{\sigma}_{h^n_M h^n_M} &= \frac{[(h^n - h^n_k)^2]^k_1}{k-1}\\
\hat{\sigma}_{x^n_M y^n_M} &= \frac{[(x^n - x^n_k)(y^n - y^n_k)]^k_1}{k-1}\\
\hat{\sigma}_{x^n_M h^n_M} &= \frac{[(x^n - x^n_k)(h^n - h^n_k)]^k_1}{k-1}\\
\hat{\sigma}_{y^n_M h^n_M} &= \frac{[(y^n - y^n_k)(h^n - h^n_k)]^k_1}{k-1}
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (2.9a)$$

From (2.4) should follow that:

$$\sigma_{x^n x^m} = 0, \qquad \sigma_{x^n y^m} = 0 \text{ etc.} \ldots\ldots\ldots\ldots \quad (2.9b)$$

and therefore the elements of $\left(\hat{\sigma}_{x^i_M x^j_M}\right)$, referring to different model points, are assumed to be zero.

Table 2.1 gives the elements of $\left(\hat{\sigma}_{x^i_M x^j_M}\right)$ of experiment I in square microns for the points 1 to 8 as far as they are not zero.

On the other hand the covariance matrix $\left(\sigma_{x^i_M x^j_M}\right)$ can be computed according to formula (2.4) introducing the predeterminated standard deviations of (2.5). The elements of this covariance matrix are given in table 2.2, leaving out the zero-elements.

The covariance matrix $\left(\hat{\sigma}_{x^i_M x^j_M}\right)$ is an estimate of $\left(\sigma_{x^i_M x^j_M}\right)$.

Sub-matrices of $\left(\hat{\sigma}^2_{x^i_M x^j_M}\right)$ and $\left(\sigma^2_{x^i_M x^j_M}\right)$ represent point standard ellipsoids. The shape and position of these ellipsoids can be presented and easily compared in diagrams showing their projections on planes parallel to:

Table 2.2. The elements of the covariance matrix $\left(\sigma_{x^i_M x^j_M}\right)$ of the points used in experiment I.

| | $\sigma_{x^n_M x^n_M}$ | $\sigma_{y^n_M y^n_M}$ | $\sigma_{h^n_M h^n_M}$ | $\sigma_{x^n_M y^n_M}$ | $\sigma_{x^n_M h^n_M}$ | $\sigma_{y^n_M h^n_M}$ |
|---|---|---|---|---|---|---|
| 1 | 52 | 114 | 327 | + 1 | + 41 | + 6 |
| 2 | 47 | 115 | 325 | 0 | − 2 | + 19 |
| 3 | 49 | 256 | 323 | −18 | + 27 | −214 |
| 4 | 47 | 232 | 324 | + 6 | − 11 | −196 |
| 5 | 47 | 215 | 328 | + 1 | + 1 | +181 |
| 6 | 47 | 263 | 327 | − 2 | − 2 | +220 |
| 7 | 70 | 252 | 325 | −56 | + 85 | −211 |
| 8 | 78 | 249 | 327 | −65 | −101 | +210 |

$z = 0 \quad y = 0 \quad x = 0$

These projections are ellipses. The three elements of these ellipses, the semi major axis $a$, the semi minor axis $b$ and the direction $\psi$ of the former, are given in table 2.3 respectively in table 2.4; $a$ and $b$ are expressed in microns and $\psi$ in grades.
These ellipses are drawn in figures 2.4, 2.5 and 2.6 respectively. The thin lines refer to $\left(\hat{\sigma}_{x^i_M x^j_M}\right)$ of table 2.3 and the thick lines refer to $\left(\sigma_{x^i_M x^j_M}\right)$ of table 2.4.

Table 2.3. The elements of the ellipses computed from $\left(\hat{\sigma}_{x^i_M x^j_M}\right)$ pertaining to experiment I.

| | $z = 0$ | | | $y = 0$ | | | $x = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| 1 | 10 | 7 | 1 | 22 | 7 | 193 | 22 | 10 | 192 |
| 2 | 8 | 5 | 193 | 13 | 5 | 0 | 13 | 8 | 0 |
| 3 | 17 | 7 | 15 | 15 | 7 | 184 | 22 | 6 | 146 |
| 4 | 14 | 7 | 186 | 13 | 7 | 9 | 19 | 3 | 149 |
| 5 | 16 | 6 | 153 | 27 | 6 | 176 | 27 | 6 | 27 |
| 6 | 9 | 7 | 41 | 19 | 8 | 2 | 20 | 7 | 19 |
| 7 | 10 | 6 | 9 | 14 | 6 | 198 | 16 | 8 | 167 |
| 8 | 12 | 5 | 187 | 15 | 5 | 187 | 17 | 9 | 37 |

Table 2.4. The elements of the ellipses computed from $\left(\sigma_{x^i_M x^j_M}\right)$ pertaining to the points used in experiment I.

| | $z = 0$ | | | $y = 0$ | | | $x = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| 1 | 11 | 7 | 199 | 17 | 6 | 188 | 17 | 11 | 1 |
| 2 | 11 | 8 | 5 | 18 | 6 | 20 | 17 | 11 | 7 |
| 3 | 16 | 6 | 12 | 17 | 6 | 185 | 21 | 9 | 153 |
| 4 | 15 | 7 | 184 | 18 | 6 | 18 | 21 | 9 | 156 |
| 5 | 15 | 7 | 182 | 18 | 6 | 189 | 20 | 9 | 42 |
| 6 | 16 | 7 | 17 | 18 | 6 | 20 | 21 | 9 | 47 |
| 7 | 16 | 6 | 2 | 17 | 6 | 198 | 21 | 9 | 154 |
| 8 | 15 | 6 | 199 | 17 | 6 | 199 | 21 | 9 | 46 |

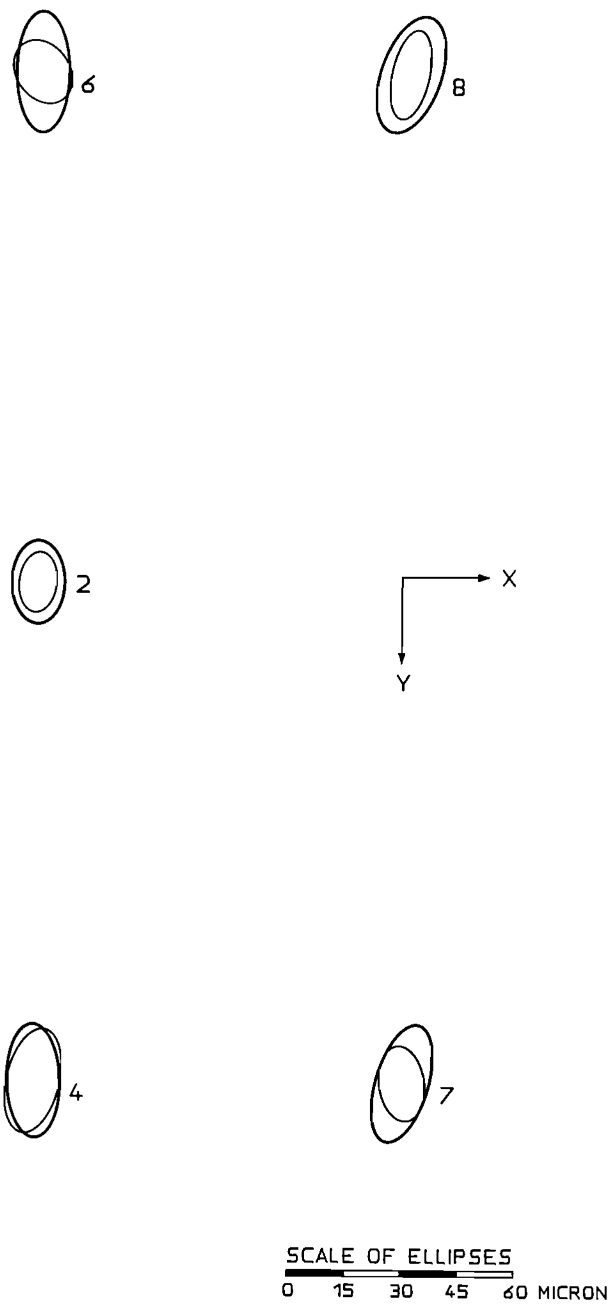


Fig. 2.4. Standard ellipses in the $xy$-plane of experiment I.

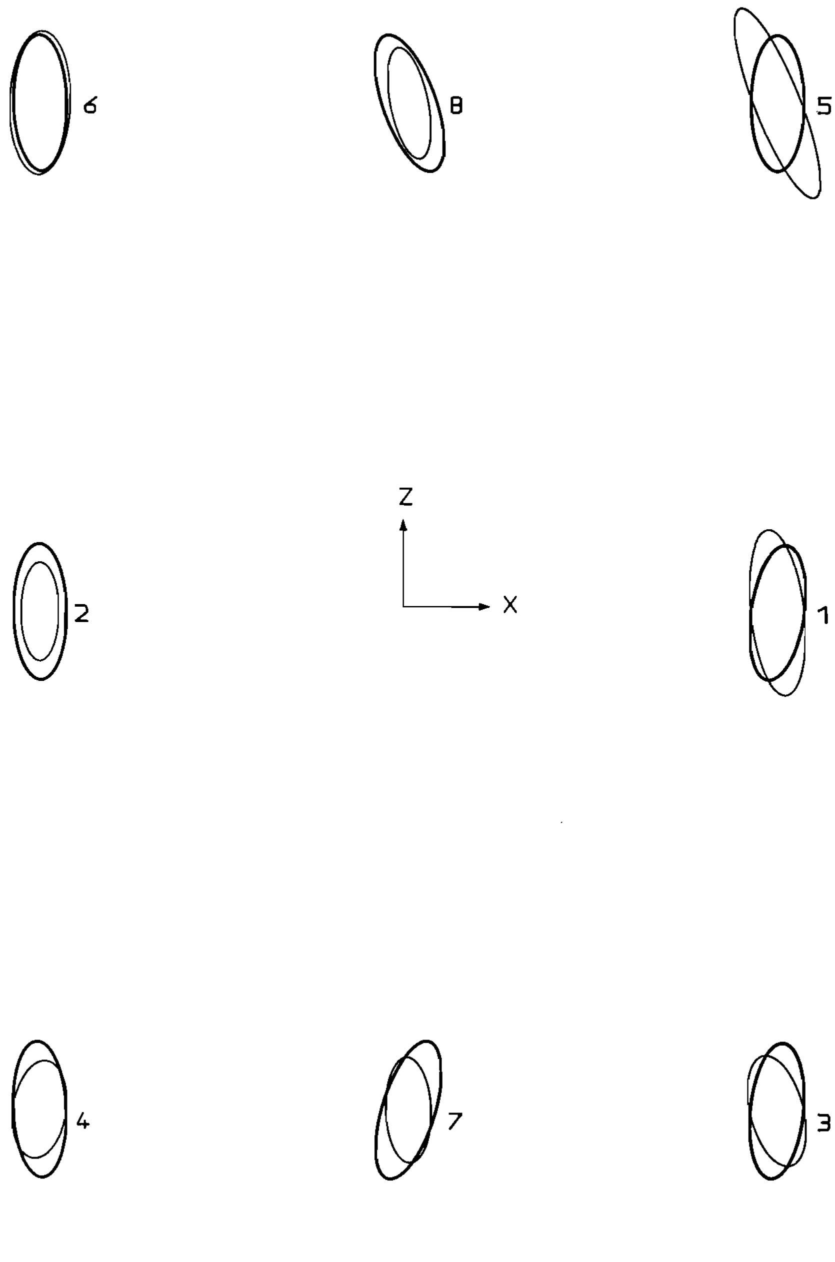


Fig. 2.5. Standard ellipses in the *xz*-plane of experiment I.

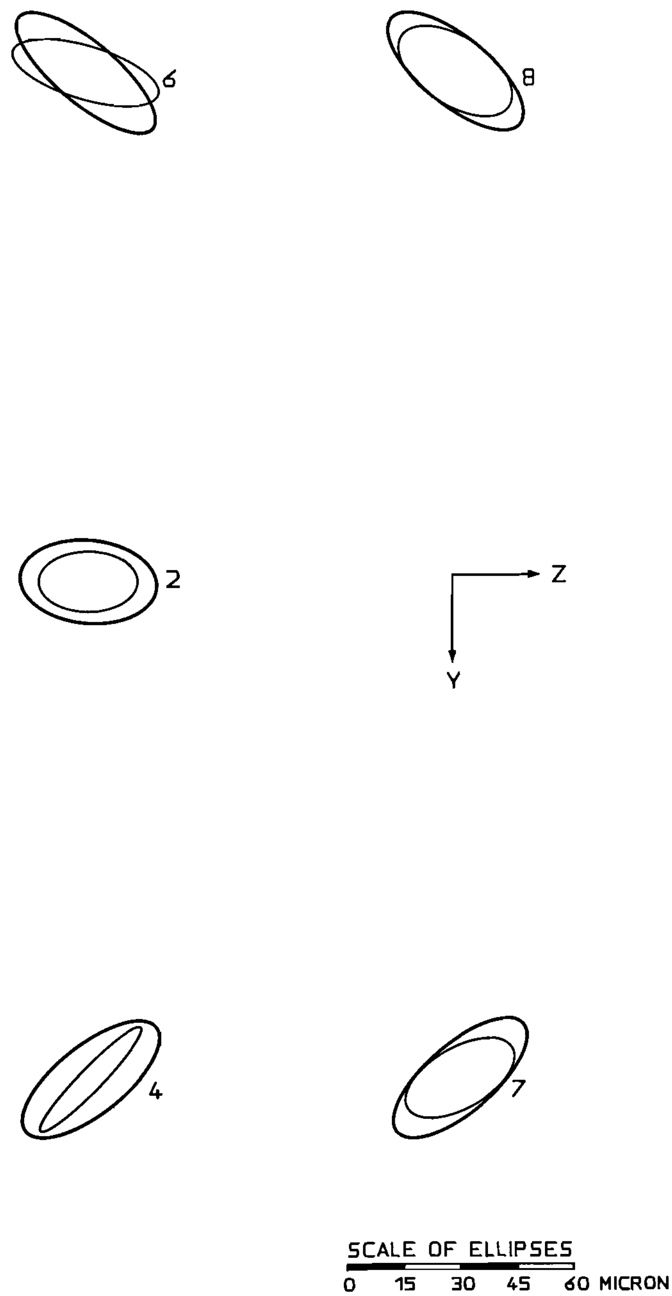


Fig. 2.6. Standard ellipses in the *yz*-plane of experiment I.

*Experiment II*
Stereopair: 147–149
Photo scale: 1:5000
Camera: Wild RC 5, $c = 152.15$ mm
Size: $23 \times 23$ cm
Distance model – projection centres: $z \simeq 250$ mm
Instrumental base: $b = 172$ mm

This experiment is just the same as the previous one. After an empirical relative orientation of this pair in a Wild A8 the machine coordinates, $x$, $y$ and $h$, of 8 pricked points are measured 20 times in a random sequence. Figure 2.7 gives the position of the 8 pricked points on two photographs and figure 2.8 shows the position in the model.

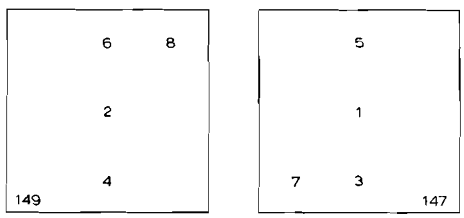


Fig. 2.7. The position of the 8 pricked points on the photographs.

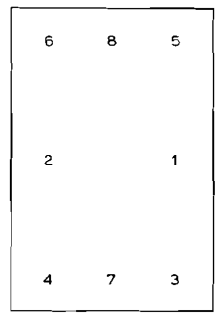


Fig. 2.8. The position of the points in the model.

Points 1, 3, 5 and 7 are pricked on the right photo 147 and points 2, 4, 6 and 8 on photo 149.
The elements of the estimated covariance matrix $\left(\sigma_{x^i_M x^j_M}\right)$ are computed from the 20 observations of each point according to formula (2.9). Table 2.5 gives these elements in square microns.
Analogous to experiment I the covariance matrix $\left(\sigma_{x^i_M x^j_M}\right)$ can be computed according to formula (2.4) introducing the standard deviations of (2.5).
The elements of this covariance matrix are given in table 2.6.
Sub-matrices of $\left(\hat{\sigma}_{x^i_M x^j_M}\right)$, the estimated covariance matrix, and $\left(\sigma_{x^i_M x^j_M}\right)$, the covariance matrix, represent point standard ellipsoids. As in the preceding part the form of the ellipsoids can easily be presented in diagrams by projections, which are ellipses, on planes parallel to:

$$z = 0 \qquad y = 0 \qquad x = 0$$

The elements of these ellipses $a$, $b$ and $\psi$ are given in tables 2.7 and 2.8 respectively for $\left(\hat{\sigma}_{x^i_M x^j_M}\right)$ and $\left(\sigma_{x^i_M x^j_M}\right)$.
The ellipses of $\left(\hat{\sigma}_{x^i_M x^j_M}\right)$ in table 2.7 and the ellipses of $\left(\sigma_{x^i_M x^j_M}\right)$ in table 2.8 are drawn in

three diagrams, figures 2.9, 2.10 and 2.11 respectively for:

$$z = 0 \qquad y = 0 \qquad x = 0$$

The thin lines refer to the estimated covariance matrix $\left(\hat{\sigma}^2_{x^i_M x^j_M}\right)$ and the thick lines refer to the covariance matrix $\left(\sigma^2_{x^i_M x^j_M}\right)$.

Table 2.5. The elements of the estimated covariance matrix $\left(\hat{\sigma}_{x^i_M x^j_M}\right)$ of experiment II.

| | $\hat{\sigma}_{x^n_M x^n_M}$ | $\hat{\sigma}_{y^n_M y^n_M}$ | $\hat{\sigma}_{h^n_M h^n_M}$ | $\hat{\sigma}_{x^n_M y^n_M}$ | $\hat{\sigma}_{x^n_M h^n_M}$ | $\hat{\sigma}_{y^n_M h^n_M}$ |
|---|---|---|---|---|---|---|
| 1 | 27 | 59 | 66 | + 9 | − 8 | − 33 |
| 2 | 68 | 179 | 79 | +11 | +42 | + 16 |
| 3 | 25 | 88 | 85 | + 4 | − 8 | − 50 |
| 4 | 51 | 127 | 163 | +34 | − 9 | −112 |
| 5 | 36 | 48 | 119 | +18 | + 6 | + 29 |
| 6 | 11 | 108 | 109 | + 5 | 0 | + 83 |
| 7 | 88 | 136 | 79 | − 5 | +29 | − 54 |
| 8 | 69 | 173 | 99 | − 9 | −35 | + 76 |

Table 2.6. The elements of the covariance matrix $\left(\sigma_{x^i_M x^j_M}\right)$ of the points used in experiment II.

| | $\sigma_{x^n_M x^n_M}$ | $\sigma_{y^n_M y^n_M}$ | $\sigma_{h^n_M h^n_M}$ | $\sigma_{x^n_M y^n_M}$ | $\sigma_{x^n_M h^n_M}$ | $\sigma_{y^n_M h^n_M}$ |
|---|---|---|---|---|---|---|
| 1 | 48 | 114 | 241 | 0 | +10 | − 7 |
| 2 | 48 | 114 | 241 | 0 | + 2 | 0 |
| 3 | 58 | 215 | 241 | −32 | +49 | −157 |
| 4 | 48 | 224 | 241 | + 6 | − 9 | −163 |
| 5 | 48 | 186 | 241 | + 2 | + 4 | +132 |
| 6 | 48 | 207 | 241 | − 3 | − 5 | +150 |
| 7 | 77 | 218 | 241 | −55 | +83 | −159 |
| 8 | 77 | 215 | 241 | −54 | −83 | +155 |

Table 2.7. The elements of the ellipses computed from $\left(\hat{\sigma}_{x^i_M x^j_M}\right)$ pertaining to experiment II.

| | $z = 0$ | | | $y = 0$ | | | $x = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| 1 | 8 | 5 | 17 | 8 | 5 | 187 | 10 | 5 | 153 |
| 2 | 13 | 8 | 6 | 11 | 6 | 46 | 13 | 9 | 90 |
| 3 | 9 | 5 | 4 | 9 | 5 | 191 | 12 | 6 | 149 |
| 4 | 12 | 6 | 23 | 13 | 7 | 195 | 16 | 6 | 155 |
| 5 | 8 | 5 | 39 | 11 | 6 | 5 | 11 | 6 | 22 |
| 6 | 10 | 3 | 3 | 10 | 3 | 0 | 14 | 5 | 50 |
| 7 | 12 | 9 | 194 | 10 | 7 | 50 | 13 | 7 | 135 |
| 8 | 13 | 8 | 194 | 11 | 7 | 163 | 15 | 7 | 65 |

Table 2.8. The elements of the ellipses computed from $\left(\sigma_{x^i_M x^j_M}\right)$ pertaining to the points used in experiment II.

| | $z = 0$ | | | $y = 0$ | | | $x = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| 1 | 11 | 7 | 2 | 15 | 6 | 178 | 15 | 11 | 195 |
| 2 | 11 | 8 | 0 | 16 | 6 | 25 | 15 | 11 | 0 |
| 3 | 15 | 6 | 7 | 15 | 6 | 190 | 19 | 9 | 150 |
| 4 | 15 | 7 | 184 | 15 | 6 | 22 | 19 | 8 | 149 |
| 5 | 14 | 7 | 183 | 15 | 6 | 177 | 18 | 9 | 46 |
| 6 | 15 | 7 | 17 | 15 | 6 | 23 | 18 | 9 | 49 |
| 7 | 15 | 6 | 0 | 15 | 6 | 0 | 19 | 9 | 150 |
| 8 | 14 | 6 | 0 | 15 | 6 | 0 | 19 | 9 | 50 |

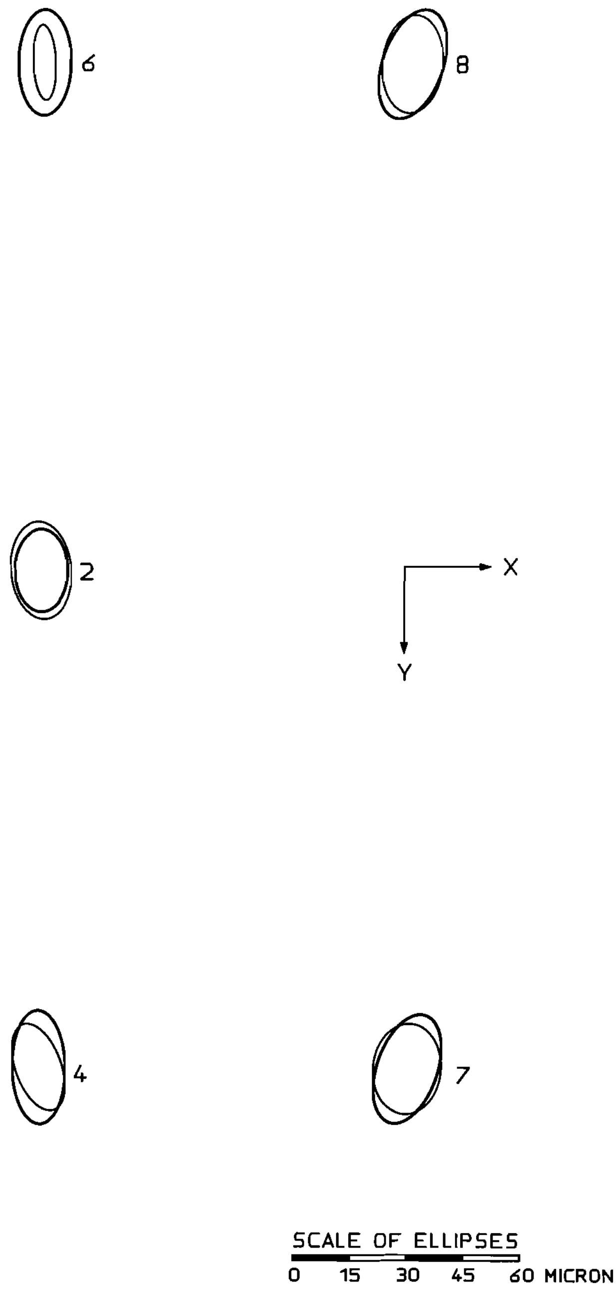


Fig. 2.9. Standard ellipses in the $xy$-plane of experiment II.

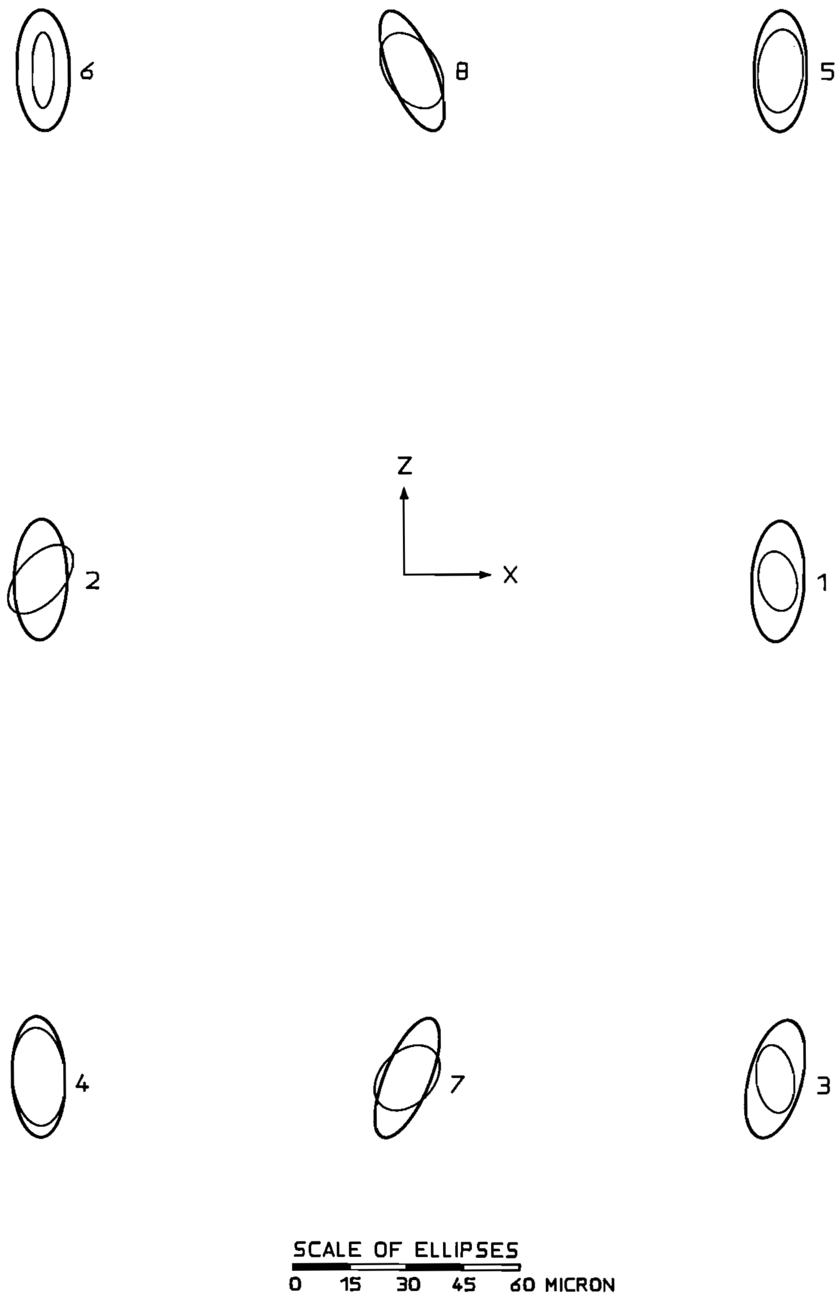


Fig. 2.10. Standard ellipses in the $xz$-plane of experiment II.

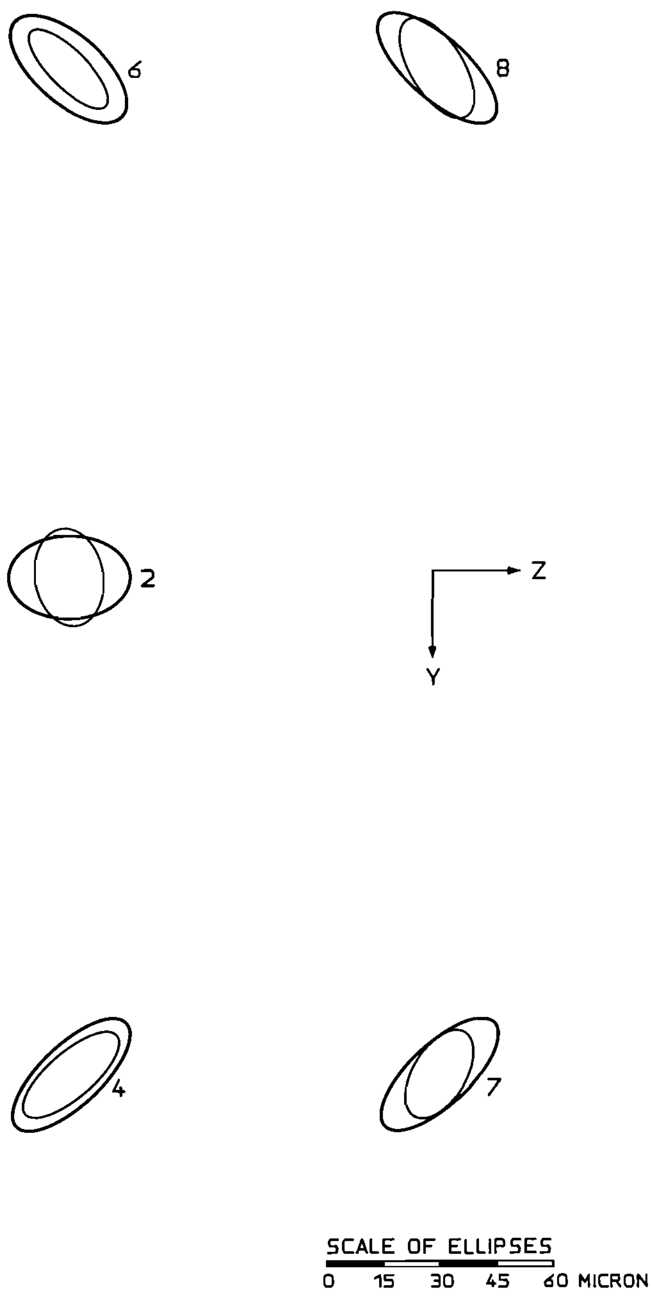


Fig. 2.11. Standard ellipses in the *yz*-plane of experiment II.

# III ERRORS DUE TO RELATIVE ORIENTATION

## 1 General description

In order to perform a relative orientation with two perspective bundles of rays $y$-parallaxes have to be measured or eliminated. The influence of the observation errors in $y$-parallaxes on the orientation elements is a known problem of which the mathematical description is given in several photogrammetric text-books. In addition the relation between the orientation elements and the machine coordinates of model points can be found in some of these text-books.

The combination of these formulae gives the influence of the error in $y$-parallax observations on the machine coordinates.

In appendix 2 a general description of these formulae is given. In that description pricked points and signalized points are distinghuished and the signs in the formulae are chosen in such a way that they can be applied to measurements made with the autograph A7 and A8.

The formulae referring to the A7 can be summarized as follows. Starting from the well-known parallax formula:

$$\Delta p_y = +\frac{y^2+z^2}{z}\Delta\omega_2 - \frac{(2x-b)y}{2z}\Delta\varphi_2 + \frac{2x-b}{2}\Delta\varkappa_2 - \frac{y}{z}\Delta bz_2 - \Delta by_2 \qquad (3.1)$$

the matrix of weight-coefficients of the orientation elements $\overline{(\Delta O),(\Delta O)^T}$ can be determined, see appendix 2:

$$\overline{(\Delta O),(\Delta O)^T} = \overline{\begin{pmatrix} \Delta\omega_2 \\ \Delta\varphi_2 \\ \Delta\varkappa_2 \\ \Delta bz_2 \\ \Delta by_2 \end{pmatrix}, \begin{pmatrix} \Delta\omega_2 \\ \Delta\varphi_2 \\ \Delta\varkappa_2 \\ \Delta bz_2 \\ \Delta by_2 \end{pmatrix}^T} \quad \ldots\ldots\ldots\ldots \quad (3.2)$$

If we write for the square of the standard deviation of the $y$-parallax observations:

$$\sigma_{p_y}^2 \quad \ldots\ldots\ldots\ldots \quad (3.3)$$

the covariance matrix of the orientation elements is:

$$(\sigma_{OO}) = \sigma_{p_y}^2 \overline{(\Delta O),(\Delta O)^T} \quad \ldots\ldots\ldots\ldots \quad (3.4)$$

The differential formula which gives the relation between the machine coordinates and the orientation elements is derived in appendix 2 and reads as follows:

$$\begin{pmatrix} \Delta x_O \\ \Delta y_O \\ \Delta y_{OL} \\ \Delta y_{OR} \\ \Delta h_O \end{pmatrix} = (A^i_O) \begin{pmatrix} \Delta\omega_2 \\ \Delta\varphi_2 \\ \Delta\varkappa_2 \\ \Delta bz_2 \\ \Delta by_2 \end{pmatrix} \quad \ldots \quad (3.5)$$

The elements of matrix $(A^i_O)$ are made up by, see appendix 2:

$x$, $y$ and $z$: the coordinates of the point $P$ concerned, the origin of the axis being in the middle of the base, see figure 2.1.

$b$: the instrumental base.

We distinguish 3 cases for the $y$-coordinate in formula (3.5):

$\Delta y_O$ : signalized points

$\Delta y_{OL}$: pricked points on left photograph

$\Delta y_{OR}$: pricked points on right photograph

The differentials of the $x$- and $h$-coordinates are the same for signalized and pricked points.

With the following denotations:

$$\begin{pmatrix} \Delta x_O \\ \Delta y_O \\ \Delta y_{OL} \\ \Delta y_{OR} \\ \Delta h_O \end{pmatrix} \equiv (\Delta x^i_O) \qquad \begin{pmatrix} \Delta\omega_2 \\ \Delta\varphi_2 \\ \Delta\varkappa_2 \\ \Delta bz_2 \\ \Delta by_2 \end{pmatrix} \equiv (\Delta O) \quad \ldots \quad (3.6)$$

(3.5) becomes:

$$(\Delta x^i_O) = (A^i_O)(\Delta O) \quad \ldots \quad (3.7)$$

The covariance matrix of the machine coordinates can be computed by application of the law of propagation of errors to (3.7):

$$\left(\sigma_{x^i_O x^j_O}\right) = (A^i_O)(\sigma_{OO})(A^j_O)^T \quad \ldots \quad (3.8)$$

in which according to (3.4):

$$(\sigma_{OO}) = \sigma^2_{p_y}\overline{(\Delta O),(\Delta O)^T}$$

The standard deviation of the $y$-parallax observations, $p_y$, is influenced by:

- the quality of the photo's
- the instrument
- the observer

etc.

From the experiments, described in the next part, the following values, given in photo scale, were derived:

$$\left.\begin{array}{l} \text{Wild A7: } \sigma_{p_y} = \ \ 9 \text{ micron} \\ \text{Wild A8: } \sigma_{p_y} = 11 \text{ micron} \end{array}\right\} \qquad (3.9)$$

Probably these differences are not caused by the type of instrument but mainly by the method of orientation, for numerical relative orientation is applied to the Wild A7 measurements and empirical relative orientation to the Wild A8 measurements.

In order to gain a better insight into the relation between the standard deviation of $y$-parallax and the photographs, the instruments, the observer, etc., a more extensive investigation would be necessary.

The covariance of the machine coordinates can be computed with (3.8). This covariance matrix describes the influence of the errors in the $y$-parallax observations for relative orientation.

In appendix 2 formulae for both A7 and A8 measurements are derived.

In the next part three experiments will be described to evaluate these formulae by practical examples.

## 2 The experiments

In order to evaluate the covariance matrix $\left(\sigma_{x_O^i x_O^j}\right)$, as described in the previous section, three experiments were executed, experiment III, IV and V.

*Experiment III*
Stereopair: 147–149
Photo scale: 1:5000
Camera: Wild RC5, $c$ = 152.15 mm
Size: 23 × 23 cm
Distance model – projection centres: $z \simeq 430$ mm
Instrumental base: $b$ = 290 mm

The relative orientation of this stereopair in a Wild A7 was made numerically by measuring the $y$-parallaxes in the well-known six points and the orientation was repeated 20 times. After each orientation the machine coordinates of 8 pricked points are measured in forward and backward sequence. Figure 3.1 gives the position of the 8 pricked points on the two photographs. Points 1, 3, 5 and 7 are pricked on the left photo 147 and 2, 4, 6 and 8 on photo 149. The position of the points in the model is given in figure 3.2.

The mean of forward and backward is called an observation. The 20 observations of each point can be considered as a sample from the probability distribution of the coordinate-variates. These variates are the three coordinates of each model point. The observations are:

$$x_k^n, y_k^n \text{ and } h_k^n \qquad (3.10)$$

$n$ = 1 to 8, number of points

$k$ = 1 to 20, number of repetitions.

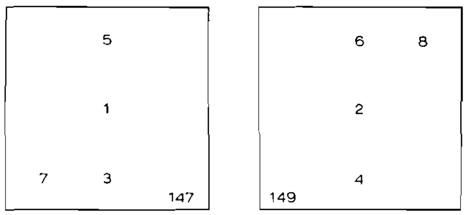


Fig. 3.1. The position of the 8 pricked points on the photographs.

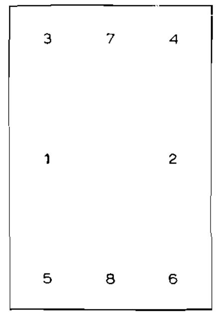


Fig. 3.2. The position of the points in the model.

The means are:

$$x^n = \frac{1}{k}[x_k^n]_1^k \qquad y^n = \frac{1}{k}[y_k^n]_1^k \qquad h^n = \frac{1}{k}[h_k^n]_1^k \qquad \ldots\ldots\ldots\ldots \quad (3.11)$$

From the differences with respect to the means an estimate for the covariance matrix of the 8 points can be computed:

$$\left(\hat{\sigma}_{x^i_{O+M} x^j_{O+M}}\right) \qquad i,j = 1,\ldots,24$$

The index $O+M$ is introduced here because this covariance matrix is caused by observation errors in both relative orientation and measuring of a model point.

The elements of this matrix are:

$$\left.\begin{aligned}
\hat{\sigma}_{x^n_{O+M} x^m_{O+M}} &= \frac{[(x^n - x_k^n)(x^m - x_k^m)]_1^k}{k-1} \\
\hat{\sigma}_{y^n_{O+M} y^m_{O+M}} &= \frac{[(y^n - y_k^n)(y^m - y_k^m)]_1^k}{k-1} \\
\hat{\sigma}_{h^n_{O+M} h^m_{O+M}} &= \frac{[(h^n - h_k^n)(h^m - h_k^m)]_1^k}{k-1} \\
\hat{\sigma}_{x^n_{O+M} y^m_{O+M}} &= \frac{[(x^n - x_k^n)(y^m - y_k^m)]_1^k}{k-1} \\
\hat{\sigma}_{x^n_{O+M} h^m_{O+M}} &= \frac{[(x^n - x_k^n)(h^m - h_k^m)]_1^k}{k-1} \\
\hat{\sigma}_{y^n_{O+M} h^m_{O+M}} &= \frac{[(y^n - y_k^n)(h^m - h_k^m)]_1^k}{k-1}
\end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots \quad (3.12)$$

These elements, computed for $n = m$ and $n \neq m$, are given in table 3.1 in square microns. This matrix, an estimate for the covariance matrix, contains $24 \times 24$ elements. As it is symmetrical, only the diagonal elements and the elements below the diagonal are given for simplicity's sake.

The covariance matrix can be computed by addition of the covariance matrix of the relative orientation, (3.8), and the covariance matrix of a model point, (2.4), the observations referring to these two covariance matrices being correlation free.

$$\left(\sigma_{x^i_{O+M}x^j_{O+M}}\right) = \left(\sigma_{x^i_O x^j_O}\right) + \tfrac{1}{2}\left(\sigma_{x^i_M x^j_M}\right) \quad \ldots\ldots\ldots\ldots \quad (3.13)$$

Introduction of (3.8) and (2.4) in (3.13) gives:

$$\left(\sigma_{x^i_{O+M}x^j_{O+M}}\right) = (A^i_O)(\sigma_{OO})(A^j_O)^T + \tfrac{1}{2}(A^i_M)(\sigma^2_M)(A^j_M)^T \quad \ldots\ldots\ldots \quad (3.14)$$

The factor $\frac{1}{2}$ in (3.13) refers to the fact that an observation is the mean of measurements in forward and backward sequence.

The elements of $\left(\sigma_{x^i_{O+M}x^j_{O+M}}\right)$ are given in table 3.2.

Sub-matrices of

$$\left(\hat{\sigma}_{x^i_{O+M}x^j_{O+M}}\right) \quad \text{and} \quad \left(\sigma_{x^i_{O+M}x^j_{O+M}}\right)$$

represent point standard ellipsoids and relative ellipsoids of which shape and position can easily be represented and compared by their projections on planes parallel to:

$$z = 0 \qquad y = 0 \qquad x = 0$$

The three elements of these projections, the ellipses, are given in tables 3.3 and 3.4; $a$, the semi major axis, and $b$, the semi minor axis, in microns and $\psi$ the direction of $a$ in grades. The standard ellipses of the points 1 to 8 are given in the upper part of the tables and in the lower part the 28 relative standard ellipses are mentioned.

The standard ellipses and some relative standard ellipses are drawn in figures 3.3, 3.4 and 3.5 referring to:

$$z = 0 \qquad y = 0 \qquad x = 0$$

respectively.

The thin lines are the ellipses of the estimated covariance, table 3.3, and the thick lines are the ellipses of the covariance matrix, table 3.4.

Table 3.1. The elements of the estimated covariance matrix $\left(\hat{\sigma}_{x^i_{O+M} x^j_{O+M}}\right)$ of experiment III.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 103 | | | | | | | | | | |
| $y_1$ | −1 | 51 | | | | | | | | | |
| $h_1$ | −835 | 210 | 12897 | | | | | | | | |
| $x_2$ | 394 | −106 | −6745 | 3876 | | | | | | | |
| $y_2$ | −87 | 2 | 299 | 72 | 784 | | | | | | |
| $h_2$ | −665 | 173 | 10471 | −5633 | 201 | 8801 | | | | | |
| $x_3$ | 152 | −29 | −1991 | 1024 | −120 | −1647 | 371 | | | | |
| $y_3$ | −304 | 129 | 5979 | −3294 | −63 | 4946 | −889 | 3083 | | | |
| $h_3$ | −592 | 187 | 10248 | −5523 | 188 | 8467 | −1635 | 5051 | 8735 | | |
| $x_4$ | 398 | −103 | −6527 | 3593 | 76 | −5271 | 982 | −3139 | −5290 | 3619 | |
| $y_4$ | −587 | 142 | 8518 | −4433 | 244 | 6750 | −1291 | 3891 | 6706 | −4482 | 6317 |
| $h_4$ | −653 | 214 | 10375 | −5716 | −79 | 8394 | −1620 | 5042 | 8683 | −5707 | 7498 |
| $x_5$ | 39 | 1 | −145 | 82 | −38 | −129 | 46 | −19 | −79 | 64 | −56 |
| $y_5$ | 546 | −112 | −7600 | 3863 | −309 | −6200 | 1170 | −3384 | −5801 | 3651 | −4842 |
| $h_5$ | −926 | 215 | 13498 | −6950 | 391 | 11068 | −2030 | 6117 | 10325 | −6672 | 8585 |
| $x_6$ | 428 | −77 | −6962 | 3881 | −90 | −5779 | 1054 | −3270 | −5476 | 3617 | −4459 |
| $y_6$ | 367 | 13 | −6628 | 3869 | 173 | −5689 | 1043 | −3236 | −5514 | 3525 | −4284 |
| $h_6$ | −651 | 52 | 9441 | −4960 | 446 | 7910 | −1519 | 4206 | 7414 | −4543 | 5929 |
| $x_7$ | 254 | −58 | −3610 | 1911 | −153 | −2965 | 599 | −1699 | −2991 | 1858 | −2408 |
| $y_7$ | −328 | 121 | 5955 | −3287 | −105 | 4922 | −900 | 3009 | 5055 | −3174 | 3887 |
| $h_7$ | −588 | 210 | 10347 | −5629 | −22 | 8532 | −1604 | 5103 | 8721 | −5388 | 6758 |
| $x_8$ | 256 | −60 | −3947 | 2183 | −70 | −3215 | 594 | −1851 | −3057 | 2074 | −2541 |
| $y_8$ | 413 | 4 | −6782 | 3821 | 124 | −5782 | 1063 | −3198 | −5490 | 3455 | −4155 |
| $h_8$ | −802 | 124 | 11097 | −5660 | 429 | 9139 | −1760 | 4825 | 8338 | −5312 | −6987 |

Table 3.2. The elements of the covariance matrix $\left(\sigma_{x^i_{O+M} x^j_{O+M}}\right)$ of the points used in experiment III.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 73.3 | | | | | | | | | | |
| $y_1$ | −3.3 | 172.4 | | | | | | | | | |
| $h_1$ | −186.0 | 187.3 | 10458.6 | | | | | | | | |
| $x_2$ | 96.4 | −97.0 | −5417.9 | 3073.8 | | | | | | | |
| $y_2$ | 3.2 | −3.2 | −180.4 | 100.2 | 578.7 | | | | | | |
| $h_2$ | −147.9 | 148.9 | 8317.0 | −4604.9 | −161.1 | 7457.5 | | | | | |
| $x_3$ | 29.9 | −30.1 | −1682.8 | 901.5 | 25.6 | −1383.8 | 404.9 | | | | |
| $y_3$ | −100.7 | 101.4 | 5661.7 | −3033.0 | −86.1 | 4655.9 | −1126.7 | 3959.8 | | | |
| $h_3$ | −162.2 | 163.2 | 9117.0 | −4884.0 | −138.6 | 7497.3 | −1814.3 | 6104.3 | 9829.7 | | |
| $x_4$ | 90.6 | −91.2 | −5094.7 | 2798.1 | 255.5 | −4295.3 | 949.3 | −3193.8 | −5143.0 | 3030.9 | |
| $y_4$ | −107.8 | 108.5 | 6061.4 | −3463.0 | −179.3 | 5316.0 | −1054.2 | 3546.8 | 5711.4 | −3403.0 | 4688.8 |
| $h_4$ | −148.8 | 149.8 | 8367.2 | −4595.3 | −419.6 | 7054.3 | −1559.0 | 5245.3 | 8446.5 | −4839.6 | 5854.0 |
| $x_5$ | −2.2 | 2.2 | 124.1 | −67.0 | −4.0 | 102.8 | −17.8 | 59.8 | 96.3 | −56.7 | 72.9 |
| $y_5$ | 101.1 | −101.7 | −5681.9 | 3068.1 | 182.1 | −4709.8 | 814.0 | −2738.8 | −4410.2 | 2595.1 | −3339.0 |
| $h_5$ | −181.2 | 182.4 | 10188.2 | −5501.4 | −326.5 | 8445.2 | −1459.7 | 4910.9 | 7908.0 | −4653.4 | 5987.3 |
| $x_6$ | 89.5 | −90.1 | −5030.3 | 2801.0 | −56.8 | −4299.8 | 742.8 | −2498.9 | −4024.1 | 2284.3 | −3039.2 |
| $y_6$ | 103.9 | −104.6 | −5841.6 | 3356.5 | 98.7 | −5152.6 | 945.9 | −3182.4 | −5124.7 | 2965.0 | −3783.1 |
| $h_6$ | −144.0 | 145.0 | 8096.0 | −4508.0 | 91.5 | 6920.2 | −1195.4 | 4021.9 | 6476.5 | −3676.5 | 4891.3 |
| $x_7$ | 48.5 | −48.8 | −2728.2 | 1470.8 | 69.6 | −2257.8 | 520.7 | −1751.9 | −2821.1 | 1557.7 | −1744.2 |
| $y_7$ | −95.0 | 95.6 | 5340.9 | −2879.3 | −136.3 | 4420.0 | −1019.4 | 3429.8 | 5522.9 | −3049.6 | 3414.6 |
| $h_7$ | −152.1 | 153.1 | 8550.9 | −4609.8 | −218.2 | 7076.5 | −1632.1 | 5491.1 | 8842.3 | −4882.4 | 5466.8 |
| $x_8$ | 45.8 | −46.1 | −2575.2 | 1409.3 | 33.4 | −2163.4 | 361.0 | −1214.4 | −1955.6 | 1140.7 | −1515.6 |
| $y_8$ | 90.7 | −91.3 | −5096.7 | 2906.7 | 94.1 | −4462.1 | 729.0 | −2452.8 | −3949.8 | 2328.7 | −3119.6 |
| $h_8$ | −147.5 | 148.4 | 8290.4 | −4536.9 | −107.6 | 6964.6 | −1162.0 | 3909.6 | 6295.7 | −3672.2 | 4879.3 |

| $h_4$ | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9868 | | | | | | | | | | | | |
| −107 | 41 | | | | | | | | | | | |
| −5753 | 137 | 4961 | | | | | | | | | | |
| 10259 | −222 | −8694 | 15570 | | | | | | | | | |
| −5491 | 104 | 4205 | −7596 | 4159 | | | | | | | | |
| −5553 | 43 | 3792 | −6792 | 3950 | 4474 | | | | | | | |
| 7001 | −121 | −5825 | 10220 | −5327 | −5386 | 7904 | | | | | | |
| −3062 | 58 | 2096 | −3695 | 1928 | 1873 | −2616 | 1090 | | | | | |
| 5209 | −43 | −3350 | 6023 | −3183 | −3215 | 4167 | −1748 | 3131 | | | | |
| 8850 | −81 | −5879 | 10482 | −5537 | −5521 | 7330 | −3017 | 5232 | 8940 | | | |
| −3115 | 76 | 2439 | −4441 | 2368 | 2091 | −2916 | 1083 | −1757 | −3065 | 1432 | | |
| −5331 | 104 | 4099 | −7281 | 4015 | 4241 | −5392 | 1903 | −3206 | −5577 | 2158 | 4377 | |
| 8277 | −206 | −7114 | 12490 | −6163 | −5719 | 8698 | −3067 | 4844 | 8626 | −3485 | −6213 | 10606 |

| $h_4$ | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8366.2 | | | | | | | | | | | | |
| 93.1 | 71.8 | | | | | | | | | | | |
| −4262.1 | −81.9 | 3921.9 | | | | | | | | | | |
| 7642.3 | 146.9 | −6729.3 | 12066.4 | | | | | | | | | |
| −3751.6 | −68.0 | 3115.9 | −5587.2 | 2983.3 | | | | | | | | |
| −4869.4 | −73.8 | 3379.4 | −6059.6 | 3253.1 | 4379.7 | | | | | | | |
| 6037.9 | 109.5 | −5014.9 | 8992.2 | −4667.8 | −5488.4 | 7916.6 | | | | | | |
| −2558.3 | −29.1 | 1332.1 | −2388.7 | 1202.3 | 1544.3 | −1935.0 | 953.4 | | | | | |
| 5008.4 | 56.9 | −2607.9 | 4676.3 | −2353.7 | −3023.2 | 3788.1 | −1729.5 | 3554.8 | | | | |
| 8018.6 | 91.2 | −4175.3 | 7486.8 | −3768.3 | −4840.2 | 6064.9 | −2768.9 | 5420.7 | 8678.6 | | | |
| −1873.4 | −36.4 | 1667.1 | −2989.3 | 1476.5 | 1600.4 | −2376.3 | 588.2 | −1151.5 | −1843.6 | 896.7 | | |
| −3824.5 | −71.0 | 3249.8 | −5827.2 | 3052.5 | 3571.8 | −4912.8 | 1191.1 | −2331.8 | −3733.3 | 1460.6 | 3695.9 | |
| 6031.0 | 117.2 | −5366.9 | 9623.4 | −4753.3 | −5152.1 | 7650.1 | −1893.6 | 3707.1 | 5935.2 | −2365.9 | −5225.7 | 8416.9 |

Table 3.3. The elements of the ellipses computed from $\left(\hat{\sigma}_{x^i_{O+M} x^j_{O+M}}\right)$ pertaining to experiment III.

| | | $z = 0$ | | | $y = 0$ | | | $x = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| | 1 | 10 | 7 | 102 | 114 | 7 | 196 | 114 | 7 | 1 |
| | 2 | 62 | 28 | 99 | 112 | 14 | 163 | 94 | 28 | 2 |
| | 3 | 58 | 10 | 182 | 95 | 8 | 188 | 108 | 11 | 34 |
| | 4 | 98 | 17 | 159 | 115 | 15 | 166 | 126 | 20 | 43 |
| | 5 | 70 | 6 | 2 | 125 | 6 | 199 | 143 | 9 | 167 |
| | 6 | 91 | 19 | 49 | 108 | 20 | 161 | 109 | 23 | 160 |
| | 7 | 64 | 9 | 167 | 100 | 8 | 179 | 110 | 7 | 34 |
| | 8 | 74 | 17 | 31 | 109 | 16 | 179 | 120 | 23 | 165 |
| 1 | 2 | 57 | 28 | 93 | 59 | 21 | 79 | 29 | 27 | 133 |
| 1 | 3 | 55 | 8 | 188 | 34 | 13 | 7 | 57 | 27 | 126 |
| 1 | 4 | 93 | 20 | 163 | 57 | 41 | 70 | 80 | 42 | 115 |
| 1 | 5 | 73 | 6 | 195 | 38 | 8 | 1 | 74 | 34 | 117 |
| 1 | 6 | 87 | 16 | 45 | 66 | 32 | 65 | 72 | 36 | 74 |
| 1 | 7 | 60 | 7 | 172 | 36 | 22 | 31 | 57 | 30 | 122 |
| 1 | 8 | 72 | 16 | 26 | 40 | 27 | 40 | 67 | 34 | 87 |
| 2 | 3 | 76 | 19 | 39 | 47 | 24 | 104 | 63 | 24 | 98 |
| 2 | 4 | 81 | 18 | 199 | 44 | 15 | 187 | 83 | 41 | 87 |
| 2 | 5 | 94 | 35 | 161 | 67 | 39 | 68 | 88 | 30 | 129 |
| 2 | 6 | 70 | 16 | 3 | 31 | 14 | 180 | 70 | 30 | 99 |
| 2 | 7 | 70 | 18 | 28 | 34 | 26 | 108 | 65 | 24 | 90 |
| 2 | 8 | 74 | 21 | 179 | 36 | 28 | 162 | 71 | 32 | 111 |
| 3 | 4 | 53 | 29 | 143 | 47 | 32 | 126 | 47 | 25 | 57 |
| 3 | 5 | 123 | 9 | 192 | 61 | 17 | 5 | 127 | 49 | 120 |
| 3 | 6 | 127 | 19 | 24 | 49 | 43 | 97 | 119 | 42 | 95 |
| 3 | 7 | 17 | 13 | 128 | 18 | 14 | 143 | 18 | 9 | 45 |
| 3 | 8 | 119 | 17 | 10 | 48 | 24 | 189 | 118 | 47 | 103 |
| 4 | 5 | 155 | 21 | 176 | 73 | 56 | 28 | 148 | 63 | 115 |
| 4 | 6 | 139 | 23 | 1 | 64 | 16 | 182 | 140 | 60 | 93 |
| 4 | 7 | 45 | 25 | 165 | 36 | 28 | 157 | 47 | 24 | 61 |
| 4 | 8 | 140 | 18 | 189 | 64 | 27 | 185 | 138 | 63 | 102 |
| 5 | 6 | 63 | 43 | 105 | 76 | 36 | 57 | 63 | 29 | 162 |
| 5 | 7 | 125 | 10 | 184 | 60 | 30 | 13 | 126 | 50 | 118 |
| 5 | 8 | 39 | 31 | 139 | 47 | 18 | 52 | 41 | 26 | 152 |
| 6 | 7 | 122 | 22 | 16 | 47 | 37 | 186 | 119 | 45 | 94 |
| 6 | 8 | 29 | 19 | 100 | 36 | 26 | 36 | 37 | 11 | 171 |
| 7 | 8 | 118 | 19 | 1 | 49 | 17 | 188 | 118 | 48 | 101 |

Table 3.4. The elements of the ellipses computed from $\left(\sigma_{x^i_{O+M} x^j_{O+M}}\right)$ pertaining to the points used in experiment III.

| | | $z = 0$ | | | $y = 0$ | | | $x = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| | 1 | 13 | 9 | 198 | 102 | 8 | 199 | 102 | 13 | 1 |
| | 2 | 55 | 24 | 97 | 102 | 13 | 164 | 86 | 24 | 199 |
| | 3 | 65 | 9 | 182 | 101 | 8 | 188 | 117 | 11 | 36 |
| | 4 | 86 | 19 | 158 | 106 | 13 | 166 | 113 | 20 | 40 |
| | 5 | 63 | 8 | 199 | 110 | 8 | 1 | 126 | 11 | 167 |
| | 6 | 84 | 19 | 43 | 104 | 13 | 165 | 109 | 20 | 160 |
| | 7 | 66 | 9 | 171 | 98 | 8 | 180 | 110 | 11 | 36 |
| | 8 | 66 | 17 | 26 | 95 | 15 | 182 | 109 | 18 | 164 |
| 1 | 2 | 55 | 27 | 95 | 57 | 31 | 76 | 36 | 27 | 7 |
| 1 | 3 | 65 | 12 | 184 | 45 | 20 | 194 | 64 | 44 | 85 |
| 1 | 4 | 84 | 22 | 158 | 55 | 45 | 85 | 68 | 46 | 104 |
| 1 | 5 | 66 | 12 | 197 | 46 | 12 | 1 | 69 | 42 | 125 |
| 1 | 6 | 85 | 21 | 41 | 55 | 45 | 76 | 69 | 46 | 91 |
| 1 | 7 | 65 | 13 | 172 | 45 | 30 | 196 | 60 | 45 | 95 |
| 1 | 8 | 68 | 18 | 23 | 48 | 29 | 8 | 64 | 48 | 103 |
| 2 | 3 | 75 | 26 | 29 | 48 | 41 | 185 | 73 | 40 | 72 |
| 2 | 4 | 75 | 23 | 199 | 44 | 17 | 176 | 76 | 39 | 88 |
| 2 | 5 | 82 | 25 | 154 | 63 | 44 | 61 | 73 | 37 | 138 |
| 2 | 6 | 69 | 21 | 1 | 41 | 17 | 177 | 70 | 38 | 111 |
| 2 | 7 | 69 | 27 | 20 | 47 | 29 | 173 | 69 | 40 | 77 |
| 2 | 8 | 68 | 25 | 176 | 45 | 33 | 181 | 66 | 41 | 121 |
| 3 | 4 | 43 | 36 | 151 | 39 | 36 | 88 | 49 | 20 | 54 |
| 3 | 5 | 117 | 14 | 190 | 78 | 22 | 197 | 116 | 77 | 110 |
| 3 | 6 | 125 | 30 | 17 | 73 | 38 | 177 | 122 | 67 | 89 |
| 3 | 7 | 26 | 17 | 185 | 29 | 17 | 185 | 35 | 15 | 45 |
| 3 | 8 | 112 | 23 | 4 | 77 | 19 | 187 | 113 | 74 | 92 |
| 4 | 5 | 134 | 25 | 175 | 72 | 57 | 196 | 126 | 67 | 115 |
| 4 | 6 | 129 | 38 | 200 | 73 | 18 | 169 | 129 | 65 | 98 |
| 4 | 7 | 40 | 27 | 172 | 33 | 28 | 162 | 45 | 20 | 58 |
| 4 | 8 | 123 | 33 | 187 | 74 | 30 | 174 | 121 | 69 | 103 |
| 5 | 6 | 57 | 39 | 95 | 61 | 38 | 68 | 54 | 25 | 156 |
| 5 | 7 | 116 | 16 | 184 | 76 | 33 | 196 | 115 | 73 | 115 |
| 5 | 8 | 36 | 29 | 156 | 42 | 22 | 45 | 44 | 20 | 152 |
| 6 | 7 | 120 | 33 | 11 | 73 | 27 | 172 | 119 | 66 | 94 |
| 6 | 8 | 31 | 29 | 49 | 33 | 30 | 34 | 40 | 18 | 152 |
| 7 | 8 | 109 | 26 | 198 | 75 | 17 | 182 | 109 | 72 | 98 |

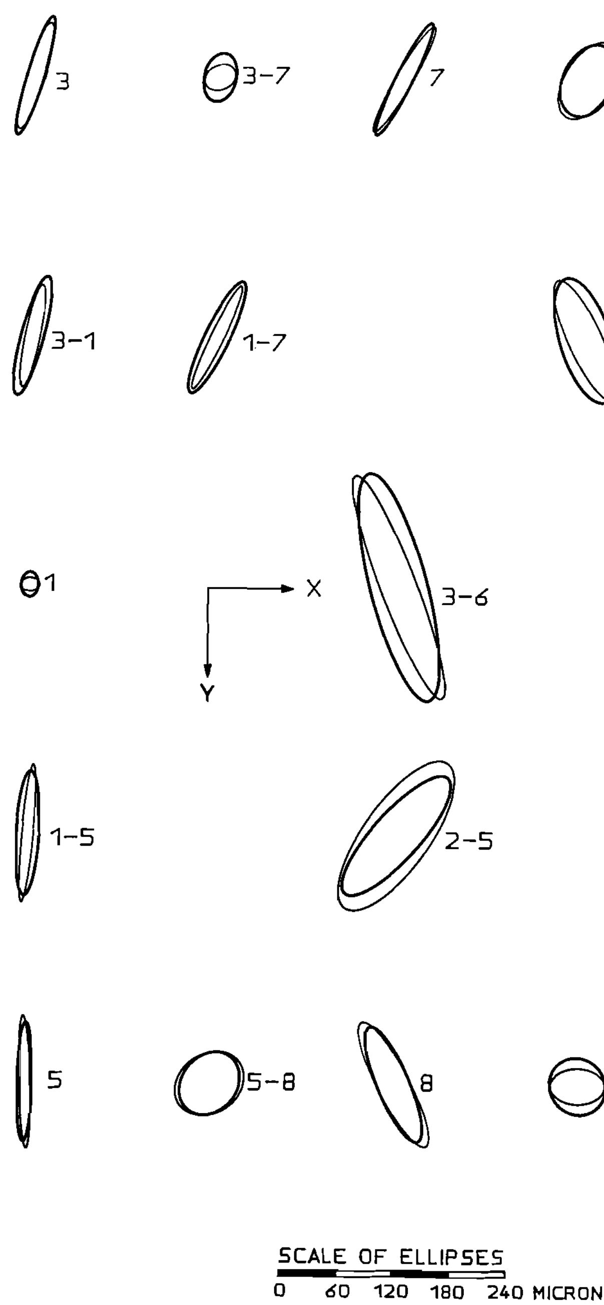


Fig. 3.3. Standard ellipses and relative standard ellipses in the *xy*-plane of experiment III.

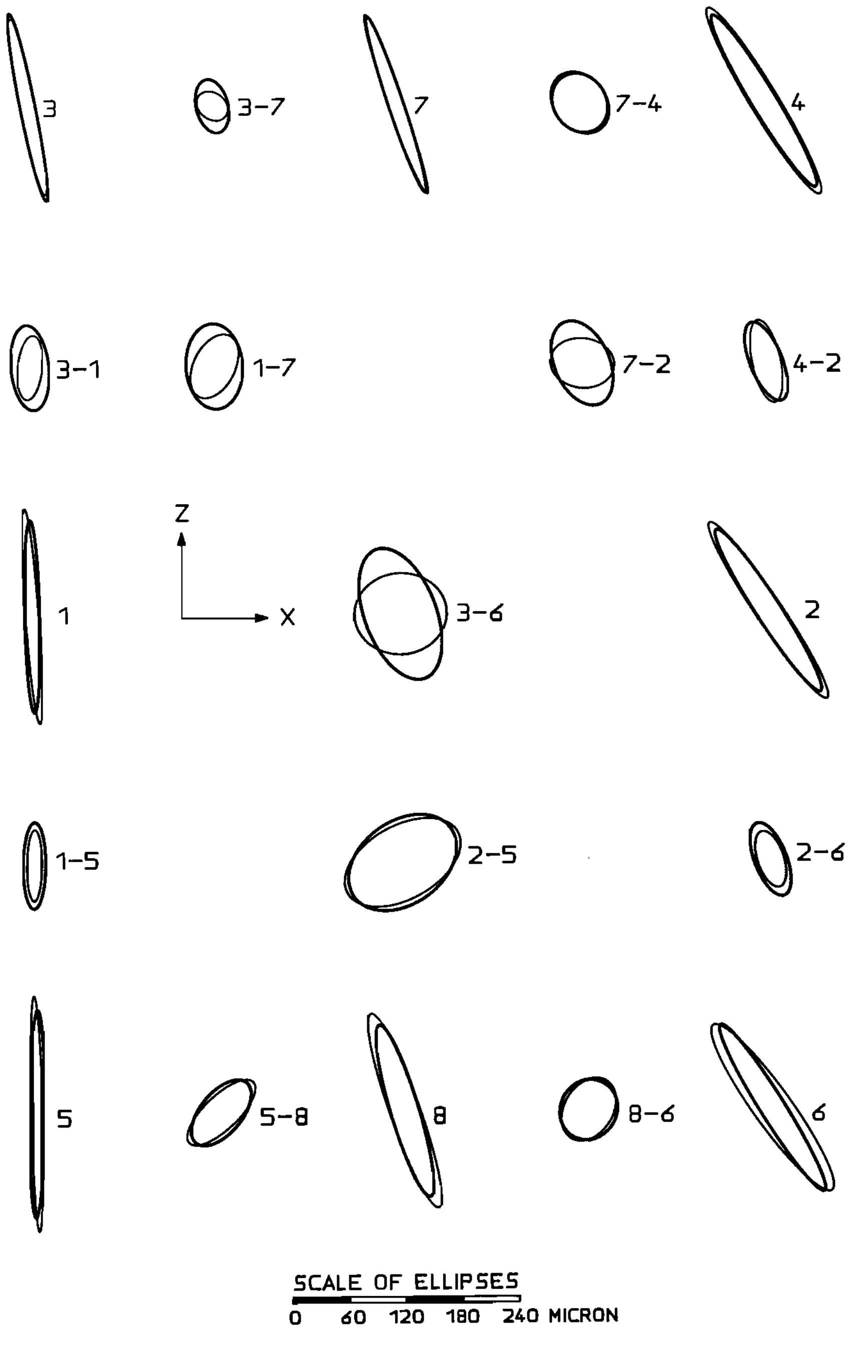


Fig. 3.4. Standard ellipses and relative standard ellipses in the $xz$-plane of experiment III.

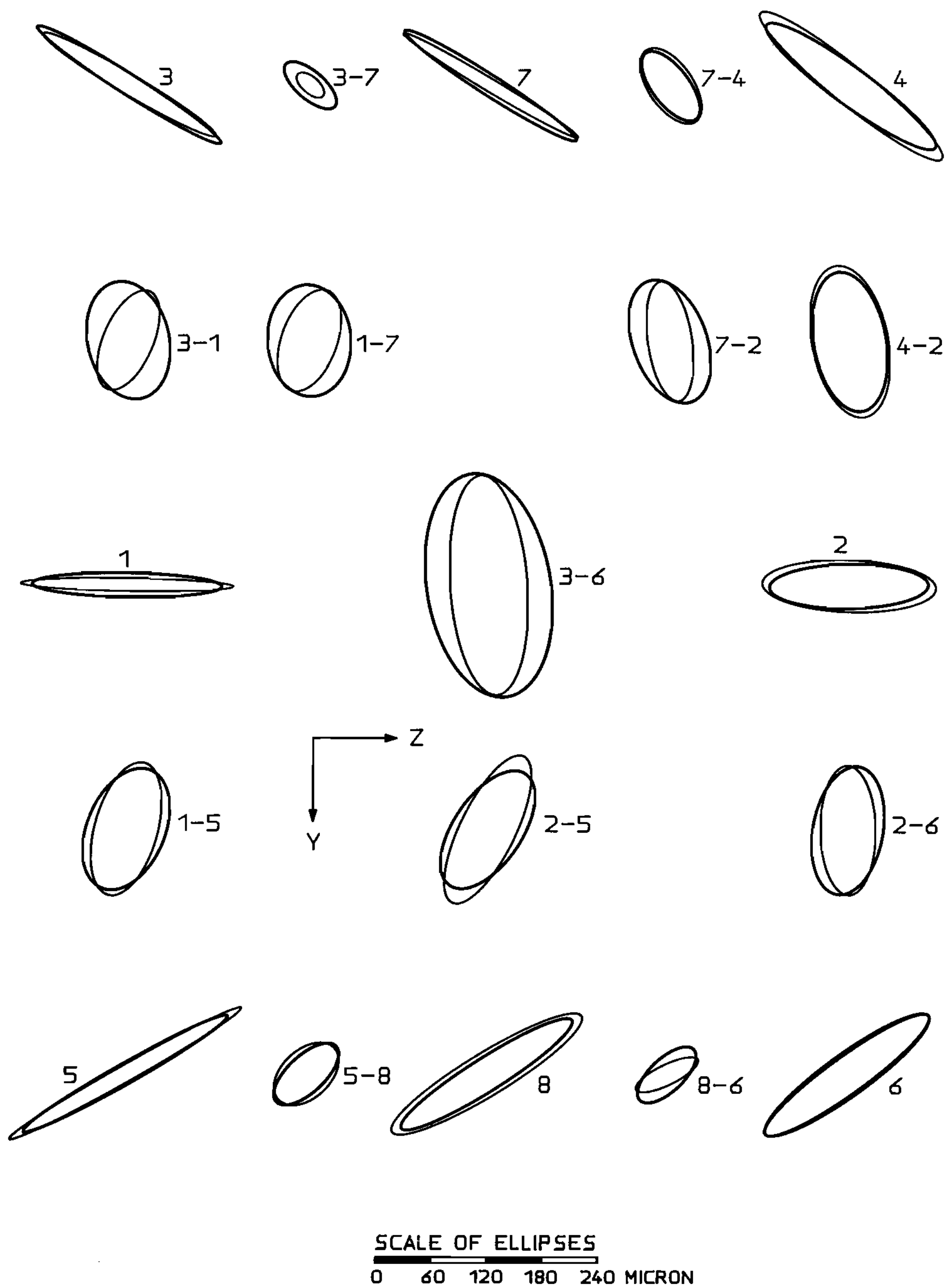


Fig. 3.5. Standard ellipses and relative standard ellipses in the $yz$-plane of experiment III.

*Experiment IV*
Stereopair: 1887–1888
Photo scale: 1:15000
Camera: Wild RC5, $c = 152.47$
Size: $23 \times 23$ cm
Distance model – projection centres: $z \simeq 307$ mm
Instrumental base: $b = 178$ mm

This experiment is similar to the previous one, experiment III. The only difference is the use of a different stereopair and a different instrument. The relative orientation of this pair was repeated 20 times in a Wild A8 and after each orientation the machine coordinates of 8 pricked points were measured in forward and backward sequence. It is evident that with this instrument the relative orientations had to be made empirically.

The position of the 8 pricked points on the photographs is given in figure 3.6; for the position in the model see figure 3.7.

Points 2, 4, 6 and 8 are pricked on photo 1888 and 1, 3, 5, and 7 on photo 1887.

The elements of the estimated covariance matrix $\left(\hat{\sigma}_{x^i_{O+M}x^j_{O+M}}\right)$ are computed from the 20 observations of the 8 points according to (3.12) and given in table 3.5 in square microns.

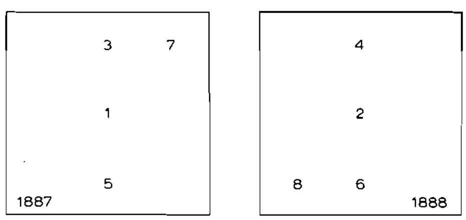


Fig. 3.6. The position of the 8 pricked points on the photographs.

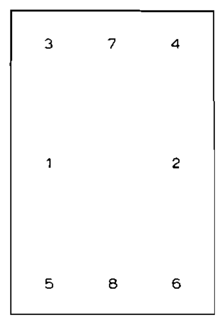


Fig. 3.7. The position of the points in the model.

Just as in experiment III the covariance matrix $\left(\sigma_{x^i_{O+M}x^j_{O+M}}\right)$ can be computed by means of (3.14) taking into account that these measurements were made with an A8 and not with an A7. The elements of this matrix are given in table 3.6.

As in the preceding experiment the shape and position of the ellipsoids are represented by their projections: ellipses. The elements of these ellipses are given in table 3.7 and table 3.8 for:

$$\left(\hat{\sigma}_{x^i_{O+M}x^j_{O+M}}\right) \quad \text{and} \quad \left(\sigma_{x^i_{O+M}x^j_{O+M}}\right)$$

The graphical representation is in figures 3.8, 3.9 and 3.10; thin lines refer to $\left(\hat{\sigma}_{x^i_{O+M}x^j_{O+M}}\right)$ and thick lines refer to $\left(\sigma_{x^i_{O+M}x^j_{O+M}}\right)$.

Table 3.5. The elements of the estimated covariance matrix $\left(\hat{\sigma}_{x^i_{O+M}x^j_{O+M}}\right)$ of experiment IV.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ | $h_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 2162 | | | | | | | | | | | |
| $y_1$ | 408 | 530 | | | | | | | | | | |
| $h_1$ | 3853 | 1253 | 15194 | | | | | | | | | |
| $x_2$ | 73 | −99 | −3315 | 1512 | | | | | | | | |
| $y_2$ | −389 | −280 | −1702 | 267 | 658 | | | | | | | |
| $h_2$ | 4521 | 1548 | 13758 | −2176 | −1512 | 14624 | | | | | | |
| $x_3$ | 1209 | −373 | 112 | 675 | 589 | 1331 | 2624 | | | | | |
| $y_3$ | 2605 | 1034 | 10150 | −2163 | −1029 | 9771 | 259 | 7851 | | | | |
| $h_3$ | 4248 | 1518 | 15451 | −2970 | −2036 | 14929 | 283 | 11134 | 17001 | | | |
| $x_4$ | −531 | −890 | −5339 | 1582 | 1127 | −4343 | 1862 | −3690 | −5713 | 3550 | | |
| $y_4$ | 2847 | 482 | 8073 | −1362 | −292 | 8254 | 1962 | 5837 | 8392 | −1428 | 6074 | |
| $h_4$ | 5082 | 1409 | 15145 | −2273 | −1820 | 15251 | 1568 | 10660 | 16539 | −4532 | 9169 | 17188 |
| $x_5$ | 2213 | 1169 | 4532 | 106 | −1093 | 5903 | −120 | 3247 | 5639 | −2080 | 2105 | 5866 |
| $y_5$ | −1967 | −586 | −7281 | 1557 | 759 | −7057 | −332 | −4625 | −7283 | 2287 | −4034 | −7337 |
| $h_5$ | 3435 | 1240 | 13086 | −2901 | −1163 | 12801 | 473 | 8761 | 13042 | −4213 | 7379 | 12990 |
| $x_6$ | 353 | 595 | −1680 | 1294 | −593 | −516 | −703 | −1145 | −844 | −264 | −1592 | −672 |
| $y_6$ | −2903 | −1252 | −10672 | 1990 | 1679 | −10557 | 613 | −7241 | −11506 | 4396 | −4942 | −10983 |
| $h_6$ | 4270 | 1652 | 14024 | −2497 | −1578 | 14577 | 523 | 9517 | 14641 | −4710 | 7711 | 14702 |
| $x_7$ | 716 | −563 | −1756 | 1042 | 862 | −573 | 2538 | −1042 | −1809 | 2610 | 987 | −442 |
| $y_7$ | 2086 | 536 | 8076 | −1791 | −619 | 7558 | 765 | 6099 | 8658 | −2523 | 5029 | 8506 |
| $h_7$ | 4098 | 1256 | 14303 | −2632 | −1890 | 13957 | 656 | 10121 | 15743 | −5097 | 7858 | 15506 |
| $x_8$ | 1421 | 962 | 1865 | 654 | −923 | 3113 | −512 | 1499 | 3004 | −1439 | 442 | 3115 |
| $y_8$ | −2237 | −818 | −7878 | 1526 | 1076 | −7679 | 226 | −5102 | −8098 | 2953 | −3920 | −7946 |
| $h_8$ | 3544 | 1393 | 12906 | −2653 | −1291 | 12705 | 107 | 8754 | 13119 | −4459 | 6939 | 12870 |

Table 3.6. The elements of the covariance matrix $\left(\sigma_{x^i_{O+M}x^j_{O+M}}\right)$ of the points used in experiment IV.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ | $h_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 1707.3 | | | | | | | | | | | |
| $y_1$ | 121.6 | 112.1 | | | | | | | | | | |
| $h_1$ | 2856.0 | 594.0 | 13682.4 | | | | | | | | | |
| $x_2$ | −58.6 | −161.2 | −3780.7 | 1689.1 | | | | | | | | |
| $y_2$ | 9.3 | −53.3 | −650.7 | −57.5 | 1977.4 | | | | | | | |
| $h_2$ | 3793.1 | 553.4 | 12752.0 | −2815.5 | −603.0 | 13492.8 | | | | | | |
| $x_3$ | 1632.0 | 23.2 | 1449.8 | 24.0 | 2738.3 | 2509.3 | 5718.8 | | | | | |
| $y_3$ | 1757.0 | 359.0 | 8240.5 | −2239.9 | −659.4 | 7735.0 | 671.5 | 5853.1 | | | | |
| $h_3$ | 2676.0 | 547.3 | 12380.1 | −3234.0 | −1556.1 | 11711.8 | 200.1 | 8667.8 | 13219.7 | | | |
| $x_4$ | 302.6 | −186.8 | −3524.9 | 1271.4 | 2649.4 | −2400.8 | 4287.7 | −2675.9 | −4702.3 | 5109.0 | | |
| $y_4$ | 2488.9 | 291.5 | 7503.2 | −1889.8 | 2042.5 | 7793.1 | 5288.1 | 4720.8 | 6361.6 | 1648.1 | 8297.6 | |
| $h_4$ | 3635.3 | 532.5 | 12331.5 | −2696.6 | −608.0 | 12611.9 | 2557.0 | 8289.8 | 12449.4 | −2676.4 | 8250.8 | 13415.3 |
| $x_5$ | 1390.5 | 148.0 | 2686.0 | 259.2 | −2377.8 | 3588.6 | −2197.8 | 1778.5 | 3477.0 | −2898.1 | −886.7 | 3202.6 |
| $y_5$ | −1563.2 | −301.5 | −7056.9 | 2050.2 | −134.6 | −6812.8 | −1400.4 | −3870.9 | −5722.5 | 1058.2 | −4269.6 | −6012.0 |
| $h_5$ | 3013.9 | 573.9 | 13472.1 | −3915.0 | 378.1 | 13025.5 | 2884.9 | 7386.8 | 10882.2 | −1843.2 | 8330.5 | 11511.5 |
| $x_6$ | −300.5 | −125.5 | −3727.3 | 1943.1 | −2550.8 | −2895.5 | −3816.9 | −1642.9 | −1592.1 | −2272.0 | −4931.6 | −2393.3 |
| $y_6$ | −2226.6 | −407.1 | −8509.9 | 1625.4 | 3064.2 | −8801.3 | 2584.0 | −5046.0 | −8507.5 | 5060.7 | −1303.7 | −7734.6 |
| $h_6$ | 3874.6 | 571.0 | 13084.5 | −2938.0 | −621.6 | 13724.5 | 2329.3 | 7165.8 | 10959.4 | −2219.2 | 7363.0 | 11961.7 |
| $x_7$ | 1193.9 | −58.6 | −438.7 | 515.9 | 2861.8 | 704.6 | 5445.0 | −618.0 | −1732.4 | 4843.5 | 4138.0 | 609.1 |
| $y_7$ | 1760.0 | 307.4 | 7371.4 | −2044.5 | 372.8 | 7023.6 | 2242.1 | 5040.7 | 7275.4 | −1034.3 | 5626.9 | 7653.4 |
| $h_7$ | 2901.1 | 517.6 | 11809.9 | −2890.6 | −1163.8 | 11503.1 | 1040.1 | 8100.7 | 12288.5 | −3807.5 | 6720.0 | 12218.5 |
| $x_8$ | 624.1 | 20.8 | −390.4 | 1152.5 | −2747.3 | 548.1 | −3347.2 | 156.7 | 1170.7 | −2921.6 | −3169.2 | 558.4 |
| $y_8$ | −1818.8 | −315.9 | −6928.1 | 1551.5 | 1324.9 | −7121.7 | 426.6 | −3788.4 | −6093.3 | 2533.1 | −2387.4 | −6003.3 |
| $h_8$ | 3202.9 | 532.2 | 12379.3 | −3230.2 | 21.1 | 12469.4 | 2591.1 | 6677.3 | 9990.1 | −1698.2 | 7409.0 | 10814.4 |

| | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7188 | | | | | | | | | | | | |
| 5866 | 4384 | | | | | | | | | | | |
| 7337 | −2462 | 3964 | | | | | | | | | | |
| 2990 | 4419 | −6975 | 12888 | | | | | | | | | |
| 672 | 1979 | 833 | −1696 | 2725 | | | | | | | | |
| 0983 | −4974 | 5523 | −9832 | −123 | 9341 | | | | | | | |
| 4702 | 6255 | −7553 | 13856 | −542 | −11415 | 15844 | | | | | | |
| 442 | −830 | 584 | −1098 | −637 | 2017 | −1274 | 2781 | | | | | |
| 8506 | 1878 | 3576 | 6683 | −1490 | −5131 | 6940 | −306 | 5252 | | | | |
| 5506 | 5228 | −6737 | 11885 | −774 | −10510 | 13451 | −1383 | 8204 | 15043 | | | |
| 3115 | 3431 | −943 | 1694 | 2420 | −3012 | 3324 | −906 | 535 | 2763 | 3263 | | |
| 7946 | −3305 | 4171 | −7467 | 214 | 6470 | −8367 | 1198 | −3685 | −7370 | −1804 | 4968 | |
| 2870 | 4967 | −6768 | 12571 | −1091 | −10120 | 13968 | −1437 | 6456 | 11915 | 2402 | −7548 | 12738 |

| | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 415.3 | | | | | | | | | | | | |
| 202.6 | 4382.5 | | | | | | | | | | | |
| 012.0 | −997.5 | 4265.5 | | | | | | | | | | |
| 511.5 | 1768.5 | −8000.9 | 15324.9 | | | | | | | | | |
| 393.3 | 3228.7 | 2821.6 | −5549.6 | 5789.5 | | | | | | | | |
| 734.6 | −5703.4 | 4202.5 | −7849.1 | −1698.2 | 10307.4 | | | | | | | |
| 961.7 | 3954.7 | −7529.5 | 14375.2 | −3321.5 | −9984.9 | 15552.3 | | | | | | |
| 609.1 | −2600.8 | −514.3 | 1191.4 | −3430.3 | 3707.9 | 672.6 | 5567.3 | | | | | |
| 653.4 | 470.3 | −3633.9 | 7003.2 | −2786.3 | −3096.0 | 6355.9 | 1013.5 | 5228.1 | | | | |
| 218.5 | 3228.0 | −5530.5 | 10545.1 | −1758.1 | −7824.6 | 10750.1 | −906.2 | 7298.4 | 12033.6 | | | |
| 558.4 | 4232.5 | 889.9 | −1865.8 | 4901.3 | −4253.4 | 567.7 | −3373.4 | −1221.3 | 926.1 | 5101.8 | | |
| 003.3 | −3162.6 | 3830.6 | −7244.4 | 516.3 | 6674.1 | −8162.5 | 1278.4 | −2812.2 | −5742.3 | −1645.2 | 5156.8 | |
| 814.4 | 2519.5 | −7270.8 | 13911.4 | −4385.7 | −8119.6 | 13948.7 | 1054.4 | 6182.7 | 9757.5 | −746.9 | −7358.1 | 13415.0 |

Table 3.7. The elements of the ellipses computed from $\left(\hat{\sigma}_{x^i_{O+M}x^j_{O+M}}\right)$ pertaining to experiment IV.

| | | z = 0 | | | y = 0 | | | x = 0 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| | 1 | 48 | 21 | 85 | 127 | 33 | 17 | 124 | 21 | 5 |
| | 2 | 40 | 24 | 82 | 122 | 34 | 190 | 122 | 22 | 193 |
| | 3 | 89 | 51 | 3 | 130 | 51 | 1 | 156 | 20 | 38 |
| | 4 | 82 | 54 | 173 | 136 | 47 | 181 | 150 | 30 | 33 |
| | 5 | 82 | 41 | 147 | 122 | 50 | 26 | 129 | 12 | 168 |
| | 6 | 97 | 52 | 199 | 126 | 52 | 197 | 156 | 27 | 159 |
| | 7 | 73 | 52 | 192 | 123 | 51 | 193 | 140 | 24 | 33 |
| | 8 | 78 | 46 | 164 | 115 | 52 | 15 | 132 | 19 | 165 |
| 1 | 2 | 64 | 34 | 71 | 61 | 46 | 79 | 48 | 42 | 188 |
| 1 | 3 | 83 | 43 | 179 | 49 | 35 | 113 | 80 | 35 | 91 |
| 1 | 4 | 96 | 56 | 144 | 83 | 45 | 106 | 77 | 43 | 85 |
| 1 | 5 | 78 | 41 | 180 | 48 | 41 | 61 | 75 | 43 | 95 |
| 1 | 6 | 115 | 59 | 18 | 67 | 51 | 72 | 112 | 53 | 108 |
| 1 | 7 | 75 | 51 | 163 | 59 | 40 | 96 | 69 | 40 | 97 |
| 1 | 8 | 84 | 51 | 198 | 57 | 38 | 59 | 85 | 46 | 98 |
| 2 | 3 | 105 | 47 | 16 | 53 | 41 | 115 | 105 | 37 | 87 |
| 2 | 4 | 86 | 42 | 190 | 44 | 36 | 110 | 87 | 33 | 88 |
| 2 | 5 | 86 | 38 | 136 | 76 | 42 | 112 | 56 | 43 | 113 |
| 2 | 6 | 83 | 37 | 185 | 43 | 34 | 66 | 82 | 35 | 109 |
| 2 | 7 | 85 | 45 | 11 | 49 | 40 | 132 | 86 | 40 | 88 |
| 2 | 8 | 75 | 36 | 150 | 59 | 44 | 109 | 59 | 44 | 104 |
| 3 | 4 | 54 | 42 | 56 | 50 | 33 | 105 | 55 | 17 | 64 |
| 3 | 5 | 151 | 75 | 180 | 88 | 57 | 122 | 147 | 58 | 90 |
| 3 | 6 | 178 | 82 | 2 | 82 | 60 | 99 | 178 | 59 | 96 |
| 3 | 7 | 31 | 16 | 22 | 24 | 18 | 14 | 36 | 12 | 60 |
| 3 | 8 | 154 | 79 | 188 | 83 | 59 | 108 | 153 | 55 | 91 |
| 4 | 5 | 155 | 79 | 161 | 112 | 61 | 113 | 136 | 61 | 91 |
| 4 | 6 | 162 | 76 | 186 | 83 | 60 | 94 | 159 | 60 | 97 |
| 4 | 7 | 37 | 31 | 162 | 39 | 28 | 155 | 47 | 15 | 51 |
| 4 | 8 | 150 | 79 | 169 | 99 | 64 | 109 | 139 | 61 | 89 |
| 5 | 6 | 66 | 33 | 59 | 58 | 29 | 118 | 53 | 21 | 132 |
| 5 | 7 | 138 | 78 | 170 | 95 | 63 | 114 | 128 | 63 | 93 |
| 5 | 8 | 28 | 24 | 106 | 29 | 20 | 74 | 29 | 16 | 144 |
| 6 | 7 | 158 | 82 | 197 | 82 | 63 | 97 | 158 | 63 | 99 |
| 6 | 8 | 46 | 20 | 46 | 37 | 21 | 131 | 40 | 20 | 129 |
| 7 | 8 | 138 | 81 | 179 | 89 | 63 | 105 | 133 | 61 | 93 |

Table 3.8. The elements of the ellipses computed from $\left(\sigma_{x^i_{O+M}x^j_{O+M}}\right)$ pertaining to the points used in experiment IV.

| | | z = 0 | | | y = 0 | | | x = 0 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| | 1 | 41 | 10 | 95 | 120 | 33 | 14 | 117 | 9 | 3 |
| | 2 | 45 | 41 | 188 | 119 | 32 | 186 | 116 | 44 | 197 |
| | 3 | 80 | 71 | 47 | 115 | 76 | 2 | 138 | 11 | 37 |
| | 4 | 95 | 66 | 26 | 119 | 66 | 182 | 140 | 47 | 40 |
| | 5 | 73 | 58 | 148 | 125 | 64 | 10 | 140 | 8 | 169 |
| | 6 | 104 | 72 | 179 | 129 | 69 | 181 | 152 | 51 | 158 |
| | 7 | 80 | 66 | 55 | 110 | 74 | 191 | 129 | 24 | 36 |
| | 8 | 82 | 59 | 151 | 116 | 71 | 194 | 133 | 29 | 166 |
| 1 | 2 | 60 | 46 | 90 | 59 | 41 | 99 | 47 | 41 | 90 |
| 1 | 3 | 76 | 60 | 166 | 68 | 41 | 126 | 73 | 46 | 91 |
| 1 | 4 | 89 | 78 | 181 | 79 | 49 | 99 | 89 | 48 | 91 |
| 1 | 5 | 72 | 56 | 18 | 63 | 38 | 133 | 72 | 42 | 118 |
| 1 | 6 | 107 | 89 | 15 | 90 | 55 | 108 | 107 | 53 | 111 |
| 1 | 7 | 73 | 65 | 145 | 71 | 45 | 111 | 69 | 46 | 100 |
| 1 | 8 | 78 | 74 | 32 | 76 | 47 | 113 | 77 | 48 | 106 |
| 2 | 3 | 96 | 86 | 4 | 90 | 50 | 124 | 99 | 52 | 82 |
| 2 | 4 | 81 | 63 | 23 | 66 | 40 | 109 | 79 | 40 | 94 |
| 2 | 5 | 83 | 72 | 169 | 76 | 51 | 115 | 87 | 42 | 127 |
| 2 | 6 | 80 | 58 | 182 | 60 | 39 | 109 | 80 | 36 | 115 |
| 2 | 7 | 81 | 78 | 28 | 82 | 44 | 122 | 81 | 48 | 87 |
| 2 | 8 | 71 | 63 | 150 | 69 | 41 | 119 | 69 | 41 | 119 |
| 3 | 4 | 69 | 47 | 193 | 49 | 40 | 129 | 77 | 23 | 68 |
| 3 | 5 | 134 | 120 | 187 | 128 | 69 | 127 | 134 | 82 | 106 |
| 3 | 6 | 164 | 137 | 184 | 142 | 76 | 118 | 162 | 83 | 100 |
| 3 | 7 | 32 | 20 | 6 | 26 | 20 | 197 | 38 | 15 | 59 |
| 3 | 8 | 140 | 128 | 161 | 138 | 72 | 121 | 136 | 81 | 96 |
| 4 | 5 | 145 | 123 | 5 | 126 | 72 | 114 | 146 | 74 | 108 |
| 4 | 6 | 146 | 124 | 198 | 125 | 70 | 108 | 146 | 70 | 105 |
| 4 | 7 | 49 | 29 | 181 | 37 | 25 | 151 | 55 | 18 | 66 |
| 4 | 8 | 136 | 126 | 17 | 129 | 69 | 113 | 135 | 72 | 103 |
| 5 | 6 | 79 | 61 | 5 | 61 | 46 | 98 | 86 | 29 | 129 |
| 5 | 7 | 129 | 123 | 2 | 128 | 71 | 121 | 131 | 76 | 113 |
| 5 | 8 | 44 | 29 | 175 | 37 | 24 | 54 | 48 | 20 | 135 |
| 6 | 7 | 150 | 133 | 176 | 138 | 73 | 115 | 148 | 77 | 105 |
| 6 | 8 | 47 | 31 | 21 | 36 | 29 | 149 | 53 | 20 | 135 |
| 7 | 8 | 133 | 125 | 125 | 136 | 70 | 118 | 127 | 77 | 103 |

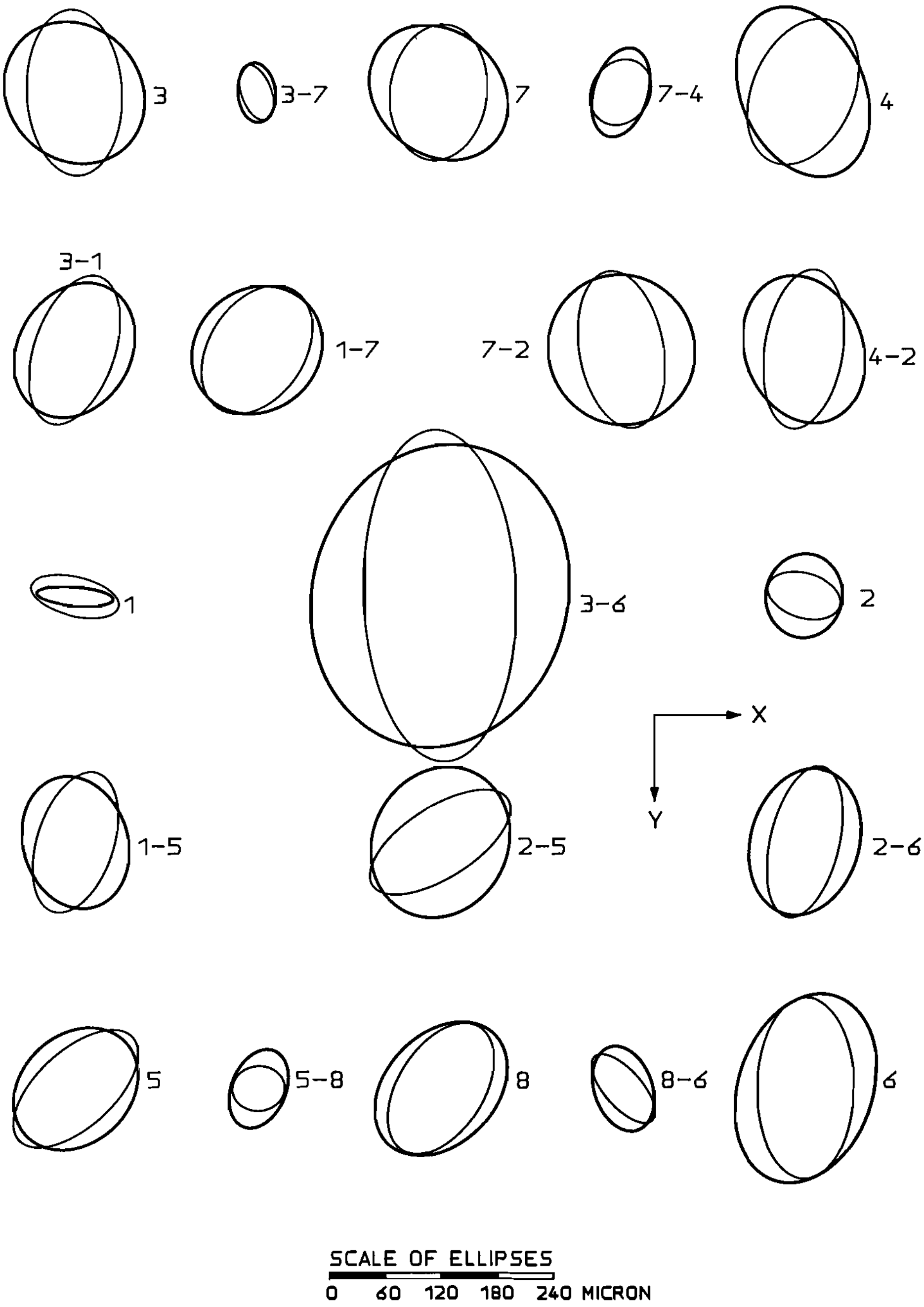


Fig. 3.8. Standard ellipses and relative standard ellipses in the $xy$-plane of experiment IV.

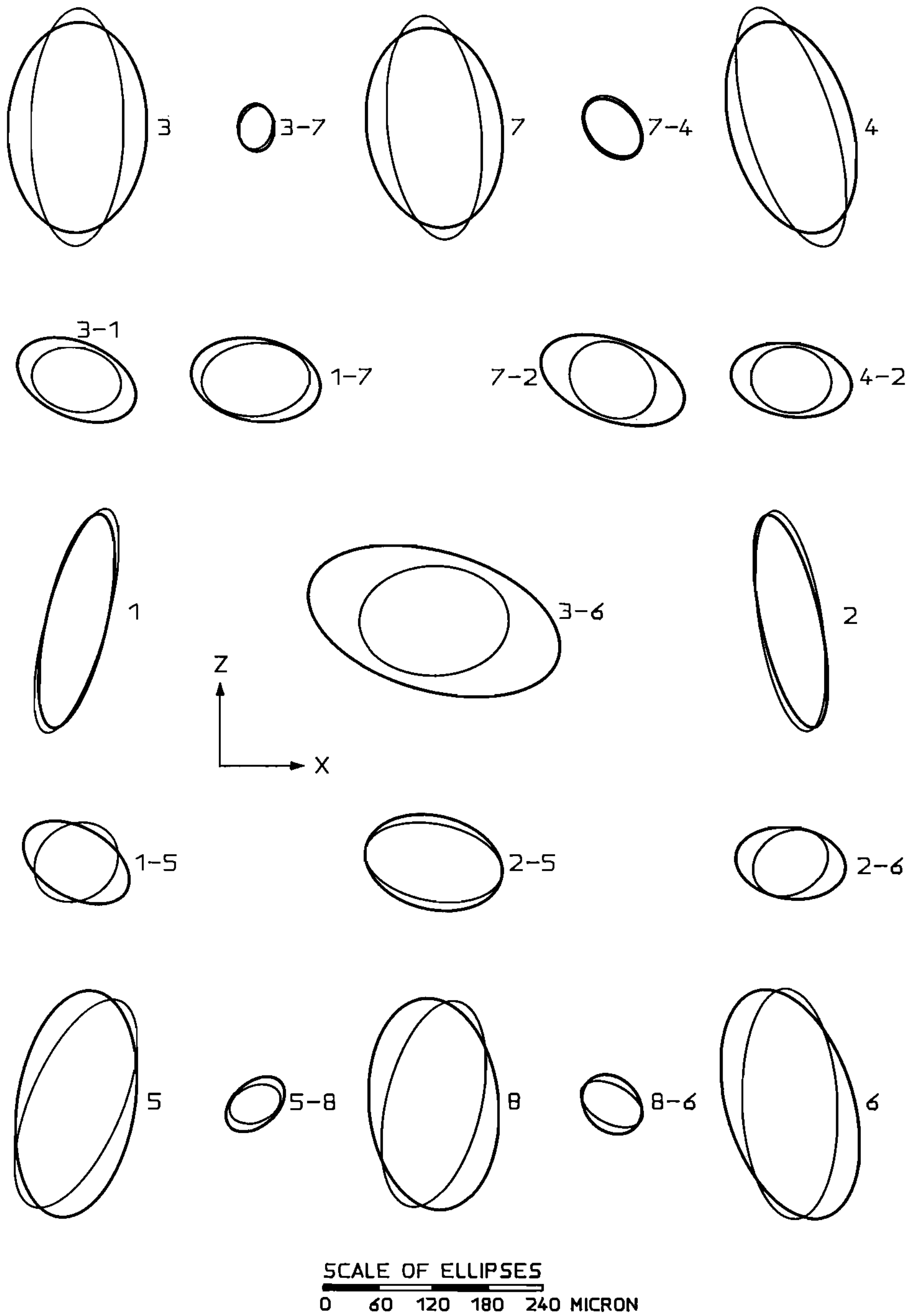


Fig. 3.9. Standard ellipses and relative standard ellipses in the *xz*-plane of experiment IV.

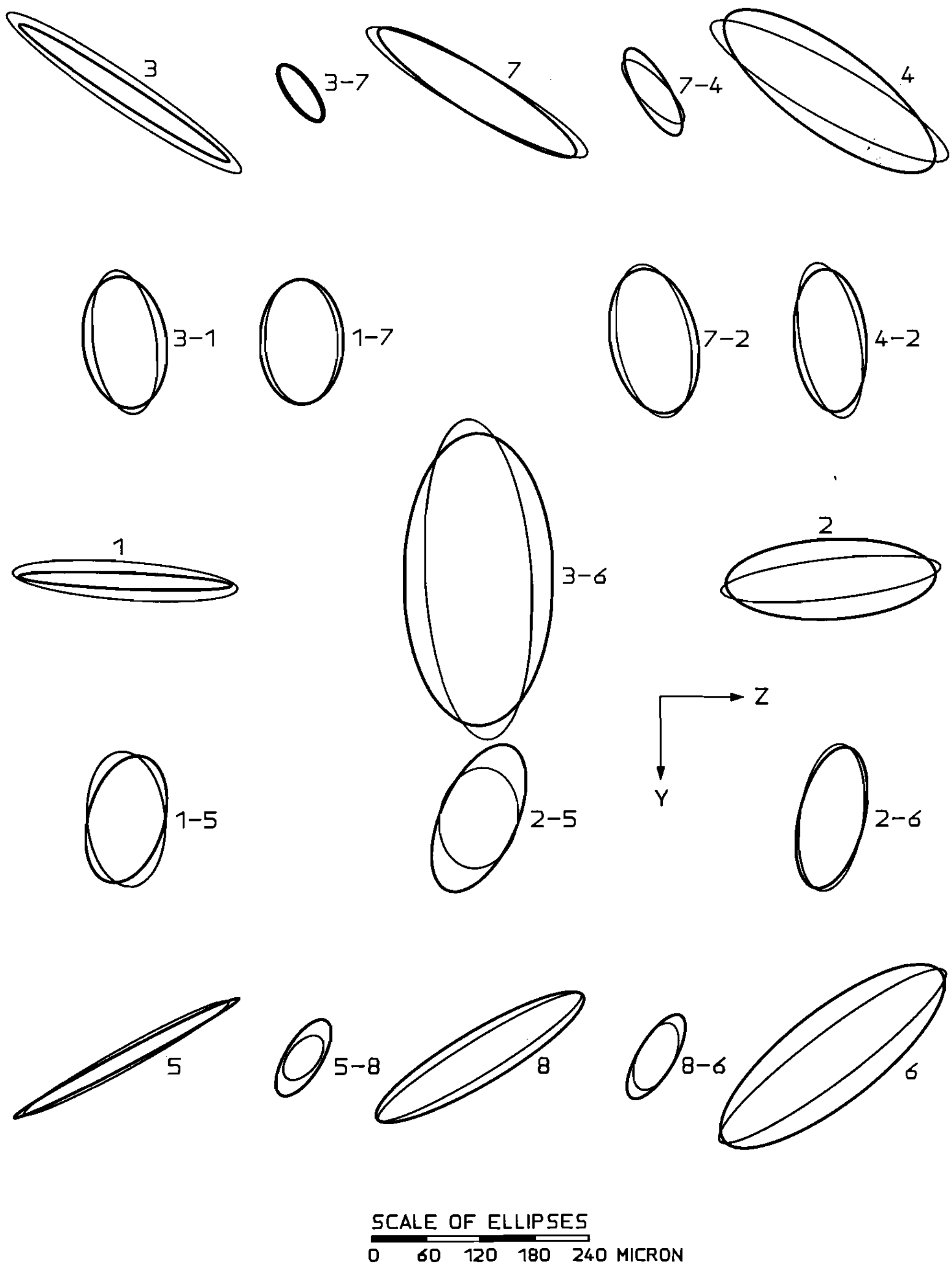


Fig. 3.10. Standard ellipses and relative standard ellipses in the $yz$-plane of experiment IV.

*Experiment V*
Stereopair: 116–118
Photo scale: 1:12000
Camera: RC5, $c = 210.38$ mm
Size: $18 \times 18$ cm
Distance model – projection centres: $z \simeq 325$ mm
Instrumental base: $b = 118$ mm

The relative orientation of this stereopair of normal angle photographs was repeated 20 times in a Wild A8 and after each relative orientation the machine coordinates of 8 signalized points were measured in forward and backward sequence. Thus the most important differences in comparison with experiment IV are: firstly normal angle instead of wide angle photo's and secondly signalized points instead of pricked points.

Figure 3.11 gives schematically the position of the eight points on the photographs and figure 3.12 the position of the points in the model.

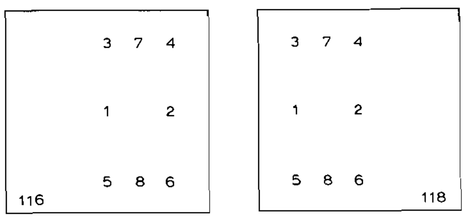


Fig. 3.11. The position of the 8 signalized points on the photographs.

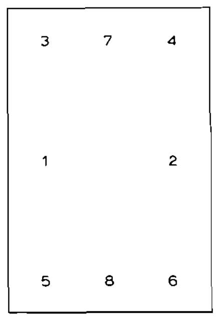


Fig. 3.12. The position of the points in the model.

The elements of the estimated covariance matrix $\left(\hat{\sigma}_{x^i_{O+M} x^j_{O+M}}\right)$ are computed from the 20 observations of each point; as in the previous experiments an observation is the mean of forward and backward. The result of the computation according to formula (3.12) is given in table 3.9. The covariance matrix $\left(\sigma_{x^i_{O+M} x^j_{O+M}}\right)$ given in table 3.10 is determined by means of formula (3.14).

The shape and position of the ellipsoids, defined by the covariance matrices, are again represented by ellipses, see figures 3.13, 3.14 and 3.15; thin lines refer to $\left(\hat{\sigma}_{x^i_{O+M} x^j_{O+M}}\right)$ and thick lines refer to $\left(\sigma_{x^i_{O+M} x^j_{O+M}}\right)$.

The elements of the ellipses are given in table 3.11 and table 3.12 respectively.

Table 3.9. The elements of the estimated covariance matrix $\left(\hat{\sigma}_{x^i_{O+M}x^j_{O+M}}\right)$ of experiment V.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ | $h_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 8396 | | | | | | | | | | | |
| $y_1$ | -614 | 474 | | | | | | | | | | |
| $h_1$ | 17410 | -2029 | 97884 | | | | | | | | | |
| $x_2$ | 2201 | 83 | -15231 | 6998 | | | | | | | | |
| $y_2$ | 1436 | 606 | 801 | 1168 | 2791 | | | | | | | |
| $h_2$ | 20402 | -2223 | 97204 | -12250 | 1665 | 98514 | | | | | | |
| $x_3$ | 10837 | -325 | 22255 | 3011 | 4335 | 26329 | 16851 | | | | | |
| $y_3$ | 5571 | -468 | 39891 | -7643 | 243 | 38721 | 7257 | 17084 | | | | |
| $h_3$ | 15342 | -2106 | 102827 | -18857 | -128 | 100581 | 19866 | 43204 | 112362 | | | |
| $x_4$ | 4334 | 326 | -12060 | 7999 | 3361 | -8085 | 7693 | -6760 | -16690 | 11029 | | |
| $y_4$ | 6799 | 73 | 29092 | -2976 | 3157 | 29977 | 11294 | 11807 | 29816 | 268 | 11927 | |
| $h_4$ | 17568 | -1819 | 87777 | -11916 | 1754 | 88484 | 23430 | 35486 | 92590 | -8087 | 27594 | 80583 |
| $x_5$ | 7205 | -1040 | 20726 | -20 | -1049 | 22559 | 6867 | 7230 | 19358 | 98 | 5024 | 18975 |
| $y_5$ | -6365 | 916 | -30199 | 3730 | -226 | -30617 | -7970 | -11695 | -30838 | 2535 | -8921 | -27206 |
| $h_5$ | 15854 | -1875 | 85801 | -12680 | 991 | 85571 | 20134 | 34390 | 88226 | -9549 | 25520 | 76275 |
| $x_6$ | 699 | -324 | -11467 | 4236 | -1410 | -9858 | -1196 | -5549 | -13351 | 3142 | -4598 | -9775 |
| $y_6$ | -5656 | 1516 | -33211 | 5448 | 2535 | -32717 | -4382 | -13105 | -34711 | 6423 | -6786 | -28643 |
| $h_6$ | 20553 | -2210 | 93396 | -10814 | 1429 | 94981 | 25661 | 36595 | 94251 | -6830 | 28322 | 84039 |
| $x_7$ | 8018 | 33 | 4224 | 6219 | 4151 | 8605 | 12991 | -260 | 272 | 10301 | 5847 | 7112 |
| $y_7$ | 6551 | -129 | 35848 | -5392 | 2211 | 35813 | 10317 | 15042 | 38083 | -2894 | 12817 | 33086 |
| $h_7$ | 16866 | -1917 | 96259 | -15327 | 1244 | 95707 | 22616 | 39709 | 103540 | -11941 | 29555 | 87882 |
| $x_8$ | 3439 | -557 | 69 | 3077 | -1058 | 1995 | 2343 | -1052 | -1831 | 2629 | -879 | 742 |
| $y_8$ | -6105 | 1322 | -31178 | 4417 | 1670 | -31206 | -5635 | -12089 | -32010 | 4663 | -7206 | -27345 |
| $h_8$ | 19288 | -2216 | 91479 | -11318 | 1073 | 92566 | 23876 | 35970 | 92506 | -7732 | 27236 | 81806 |

Table 3.10. The elements of the covariance matrix $\left(\sigma_{x^i_{O+M}x^j_{O+M}}\right)$ of the points used in experiment V.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ | $h_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 7776 | | | | | | | | | | | |
| $y_1$ | -367 | 121 | | | | | | | | | | |
| $h_1$ | 21739 | -2418 | 139571 | | | | | | | | | |
| $x_2$ | -9 | 382 | -24219 | 7623 | | | | | | | | |
| $y_2$ | -112 | -418 | 2891 | 224 | 11121 | | | | | | | |
| $h_2$ | 23923 | -2329 | 135186 | -21040 | 2669 | 133290 | | | | | | |
| $x_3$ | 8156 | -913 | 28857 | -443 | 12957 | 30700 | 24175 | | | | | |
| $y_3$ | 8678 | -738 | 53385 | -9308 | -1815 | 52060 | 8126 | 21799 | | | | |
| $h_3$ | 22137 | -2152 | 142739 | -25463 | -1761 | 138513 | 24266 | 57014 | 150655 | | | |
| $x_4$ | 285 | -98 | -17804 | 6967 | 10300 | -15065 | 11806 | -9675 | -23492 | 15718 | | |
| $y_4$ | 6311 | -982 | 40050 | -5807 | 10795 | 39164 | 20118 | 13001 | 37287 | 4893 | 20822 | |
| $h_4$ | 20634 | -1897 | 117077 | -18323 | 2059 | 115306 | 26395 | 45822 | 121515 | -13673 | 34135 | 101087 |
| $x_5$ | 8689 | -107 | 25815 | -1567 | -10086 | 27890 | -2516 | 12831 | 30478 | -10125 | -1683 | 24167 |
| $y_5$ | -6757 | 909 | -43445 | 7250 | -3523 | -42135 | -11981 | -15599 | -42639 | 2758 | -14676 | -35935 |
| $h_5$ | 19251 | -2453 | 127837 | -22354 | 6300 | 123529 | 29896 | 47252 | 127846 | -12895 | 39558 | 105851 |
| $x_6$ | 1304 | 768 | -20546 | 6254 | -11757 | -17158 | -13035 | -4465 | -16080 | -5173 | -15512 | -14415 |
| $y_6$ | -8431 | 486 | -45899 | 8028 | 9788 | -45101 | 1886 | -20190 | -50970 | 15538 | -3419 | -38885 |
| $h_6$ | 23467 | -2457 | 133954 | -20956 | 2799 | 131535 | 30134 | 50659 | 135332 | -14608 | 38421 | 112729 |
| $x_7$ | 4171 | -516 | 4041 | 3789 | 12577 | 6514 | 18987 | -1622 | -1640 | 15086 | 12990 | 5191 |
| $y_7$ | 7799 | -895 | 48564 | -7837 | 4773 | 47425 | 14806 | 18034 | 48950 | -2371 | 17650 | 41549 |
| $h_7$ | 21805 | -2060 | 132440 | -22307 | 264 | 129396 | 25944 | 52392 | 138671 | -18834 | 36501 | 113447 |
| $x_8$ | 4111 | 475 | -4330 | 3669 | -11585 | -1262 | -9617 | 1704 | 367 | -7136 | -11021 | -794 |
| $y_8$ | -7815 | 627 | -44820 | 7726 | 5254 | -43850 | -2785 | -18526 | -47872 | 11152 | -7185 | -37665 |
| $h_8$ | 22107 | -2472 | 132473 | -21543 | 4079 | 129379 | 30243 | 49687 | 133323 | -14032 | 39035 | 110849 |

| $h_4$ | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 80583 | | | | | | | | | | | | |
| 18975 | 8865 | | | | | | | | | | | |
| −27206 | −7281 | 9958 | | | | | | | | | | |
| 76275 | 18886 | −27090 | 76600 | | | | | | | | | |
| −9775 | 1223 | 2935 | −9889 | 4503 | | | | | | | | |
| −28643 | −9358 | 10684 | −29430 | 1795 | 14649 | | | | | | | |
| 84039 | 23153 | −29897 | 83453 | −8622 | −32369 | 93509 | | | | | | |
| 7112 | 3562 | −2538 | 4637 | 1240 | 1508 | 9032 | 12602 | | | | | |
| 33086 | 6000 | −10629 | 30968 | −5565 | −9818 | 33568 | 3520 | 14907 | | | | |
| 87882 | 19156 | −29267 | 82969 | −11859 | −31455 | 89949 | 4450 | 36346 | 97188 | | | |
| 742 | 4106 | −747 | 413 | 3432 | −2033 | 3014 | 2738 | −1262 | −671 | 3873 | | |
| −27345 | −8871 | 10161 | −27833 | 1733 | 13069 | −30901 | −122 | −9600 | −29627 | −1998 | 12048 | |
| 81806 | 22342 | −29241 | 81945 | −8756 | −32018 | 91049 | 7615 | 32630 | 87840 | 2532 | −30410 | 89117 |

| $h_4$ | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 101087 | | | | | | | | | | | | |
| 24167 | 19015 | | | | | | | | | | | |
| −35935 | −5716 | 14506 | | | | | | | | | | |
| 105851 | 19917 | −41322 | 119906 | | | | | | | | | |
| −14415 | 10824 | 9142 | −23328 | 18358 | | | | | | | | |
| −38885 | −19611 | 11983 | −38911 | −4814 | 25950 | | | | | | | |
| 112729 | 27436 | −42422 | 123851 | −17769 | −45093 | 131882 | | | | | | |
| 5191 | −7317 | −4403 | 7468 | −9692 | 10106 | 6509 | 18356 | | | | | |
| 41549 | 5703 | −15751 | 45132 | 10474 | −12162 | 46286 | 6044 | 18671 | | | | |
| 113447 | 27746 | −40045 | 119088 | −15632 | −45667 | 126374 | 1949 | 46296 | 128812 | | | |
| −794 | 14267 | 4043 | −8378 | 16311 | −10434 | −1815 | −9133 | −4936 | −316 | 16249 | | |
| −37665 | −14816 | 12748 | −39494 | −90 | 21062 | −43901 | 5171 | −13297 | −43509 | −5495 | 18222 | |
| 110849 | 24890 | −42244 | 123050 | −19827 | −43071 | 129606 | 6927 | 46109 | 124403 | −4169 | −42643 | 128212 |

Table 3.11. The elements of the ellipses computed from $\left(\hat{\sigma}_{x^i_{O+M} x^j_{O+M}}\right)$ pertaining to experiment V.

| | | $z = 0$ | | | $y = 0$ | | | $x = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| | 1 | 92 | 21 | 105 | 318 | 72 | 12 | 313 | 21 | 199 |
| | 2 | 85 | 50 | 84 | 316 | 73 | 192 | 314 | 53 | 1 |
| | 3 | 156 | 99 | 49 | 341 | 114 | 13 | 359 | 20 | 23 |
| | 4 | 110 | 105 | 17 | 286 | 101 | 193 | 300 | 47 | 22 |
| | 5 | 129 | 46 | 152 | 285 | 63 | 16 | 294 | 18 | 178 |
| | 6 | 122 | 65 | 11 | 307 | 61 | 194 | 324 | 55 | 178 |
| | 7 | 132 | 100 | 40 | 312 | 111 | 3 | 333 | 34 | 23 |
| | 8 | 112 | 58 | 186 | 299 | 62 | 2 | 316 | 39 | 179 |
| 1 | 2 | 105 | 44 | 107 | 105 | 45 | 100 | 56 | 31 | 51 |
| 1 | 3 | 136 | 59 | 6 | 68 | 59 | 182 | 139 | 62 | 86 |
| 1 | 4 | 138 | 63 | 153 | 111 | 37 | 75 | 112 | 51 | 111 |
| 1 | 5 | 93 | 53 | 195 | 56 | 51 | 152 | 99 | 40 | 74 |
| 1 | 6 | 138 | 68 | 49 | 107 | 68 | 103 | 111 | 67 | 92 |
| 1 | 7 | 130 | 62 | 181 | 72 | 48 | 82 | 125 | 50 | 98 |
| 1 | 8 | 111 | 55 | 34 | 75 | 62 | 77 | 100 | 62 | 90 |
| 2 | 3 | 174 | 83 | 48 | 134 | 99 | 99 | 150 | 81 | 71 |
| 2 | 4 | 93 | 43 | 10 | 49 | 42 | 155 | 96 | 36 | 121 |
| 2 | 5 | 153 | 75 | 145 | 129 | 56 | 116 | 122 | 48 | 76 |
| 2 | 6 | 112 | 54 | 193 | 55 | 45 | 112 | 111 | 45 | 96 |
| 2 | 7 | 130 | 60 | 35 | 87 | 63 | 120 | 116 | 65 | 93 |
| 2 | 8 | 116 | 52 | 172 | 69 | 49 | 111 | 108 | 48 | 90 |
| 3 | 4 | 117 | 66 | 78 | 125 | 68 | 64 | 110 | 31 | 43 |
| 3 | 5 | 225 | 109 | 1 | 114 | 107 | 161 | 233 | 93 | 81 |
| 3 | 6 | 258 | 124 | 27 | 155 | 131 | 110 | 245 | 124 | 87 |
| 3 | 7 | 61 | 40 | 76 | 67 | 38 | 61 | 63 | 20 | 45 |
| 3 | 8 | 238 | 112 | 18 | 129 | 126 | 33 | 235 | 120 | 86 |
| 4 | 5 | 218 | 110 | 169 | 141 | 67 | 94 | 200 | 67 | 96 |
| 4 | 6 | 200 | 96 | 0 | 96 | 78 | 102 | 202 | 74 | 108 |
| 4 | 7 | 58 | 30 | 76 | 62 | 35 | 63 | 54 | 16 | 40 |
| 4 | 8 | 199 | 93 | 188 | 101 | 75 | 78 | 196 | 77 | 105 |
| 5 | 6 | 105 | 56 | 92 | 109 | 47 | 121 | 58 | 56 | 146 |
| 5 | 7 | 218 | 113 | 186 | 120 | 88 | 104 | 218 | 80 | 88 |
| 5 | 8 | 68 | 41 | 93 | 71 | 36 | 125 | 47 | 36 | 155 |
| 6 | 7 | 227 | 111 | 16 | 123 | 102 | 119 | 222 | 103 | 97 |
| 6 | 8 | 39 | 23 | 94 | 40 | 20 | 120 | 26 | 20 | 53 |
| 7 | 8 | 215 | 104 | 5 | 105 | 103 | 94 | 215 | 102 | 95 |

Table 3.12. The elements of the ellipses computed from $\left(\sigma_{x^i_{O+M} x^j_{O+M}}\right)$ pertaining to the points used in experiment V.

| | | $z = 0$ | | | $y = 0$ | | | $x = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| | 1 | 88 | 10 | 103 | 378 | 65 | 10 | 374 | 9 | 199 |
| | 2 | 106 | 87 | 4 | 370 | 65 | 190 | 365 | 105 | 1 |
| | 3 | 177 | 122 | 55 | 394 | 140 | 12 | 415 | 14 | 23 |
| | 4 | 154 | 113 | 35 | 321 | 117 | 190 | 337 | 91 | 22 |
| | 5 | 151 | 103 | 138 | 352 | 123 | 12 | 366 | 15 | 179 |
| | 6 | 168 | 127 | 171 | 367 | 125 | 190 | 385 | 97 | 178 |
| | 7 | 157 | 112 | 49 | 359 | 135 | 1 | 382 | 42 | 22 |
| | 8 | 151 | 108 | 156 | 358 | 127 | 198 | 378 | 60 | 179 |
| 1 | 2 | 124 | 110 | 108 | 124 | 49 | 95 | 110 | 50 | 102 |
| 1 | 3 | 153 | 125 | 200 | 133 | 53 | 124 | 155 | 64 | 89 |
| 1 | 4 | 157 | 146 | 150 | 156 | 71 | 82 | 159 | 65 | 121 |
| 1 | 5 | 114 | 96 | 14 | 105 | 47 | 128 | 115 | 58 | 86 |
| 1 | 6 | 164 | 147 | 40 | 154 | 59 | 97 | 159 | 59 | 98 |
| 1 | 7 | 146 | 131 | 173 | 135 | 56 | 109 | 145 | 56 | 109 |
| 1 | 8 | 134 | 122 | 37 | 126 | 53 | 101 | 132 | 50 | 90 |
| 2 | 3 | 199 | 172 | 38 | 181 | 82 | 105 | 198 | 65 | 82 |
| 2 | 4 | 103 | 95 | 29 | 99 | 59 | 114 | 112 | 39 | 130 |
| 2 | 5 | 185 | 168 | 166 | 178 | 66 | 116 | 182 | 76 | 107 |
| 2 | 6 | 133 | 115 | 187 | 116 | 45 | 104 | 132 | 46 | 101 |
| 2 | 7 | 145 | 132 | 33 | 138 | 51 | 113 | 143 | 57 | 95 |
| 2 | 8 | 140 | 126 | 172 | 130 | 48 | 111 | 137 | 52 | 101 |
| 3 | 4 | 138 | 118 | 48 | 145 | 63 | 65 | 147 | 61 | 65 |
| 3 | 5 | 260 | 219 | 5 | 234 | 91 | 125 | 264 | 112 | 87 |
| 3 | 6 | 300 | 259 | 17 | 264 | 104 | 108 | 300 | 100 | 90 |
| 3 | 7 | 74 | 59 | 52 | 75 | 33 | 68 | 74 | 31 | 67 |
| 3 | 8 | 279 | 243 | 13 | 249 | 100 | 113 | 282 | 99 | 88 |
| 4 | 5 | 255 | 234 | 188 | 236 | 94 | 107 | 258 | 85 | 112 |
| 4 | 6 | 232 | 211 | 0 | 211 | 86 | 104 | 236 | 72 | 114 |
| 4 | 7 | 66 | 61 | 36 | 74 | 39 | 57 | 78 | 34 | 58 |
| 4 | 8 | 231 | 215 | 194 | 216 | 86 | 105 | 235 | 75 | 113 |
| 5 | 6 | 128 | 125 | 195 | 127 | 61 | 110 | 135 | 48 | 122 |
| 5 | 7 | 254 | 228 | 195 | 236 | 81 | 118 | 254 | 103 | 100 |
| 5 | 8 | 87 | 80 | 167 | 83 | 44 | 110 | 90 | 35 | 123 |
| 6 | 7 | 263 | 236 | 8 | 239 | 84 | 109 | 263 | 89 | 99 |
| 6 | 8 | 47 | 42 | 45 | 45 | 28 | 116 | 49 | 22 | 129 |
| 7 | 8 | 252 | 230 | 2 | 234 | 81 | 112 | 252 | 91 | 99 |

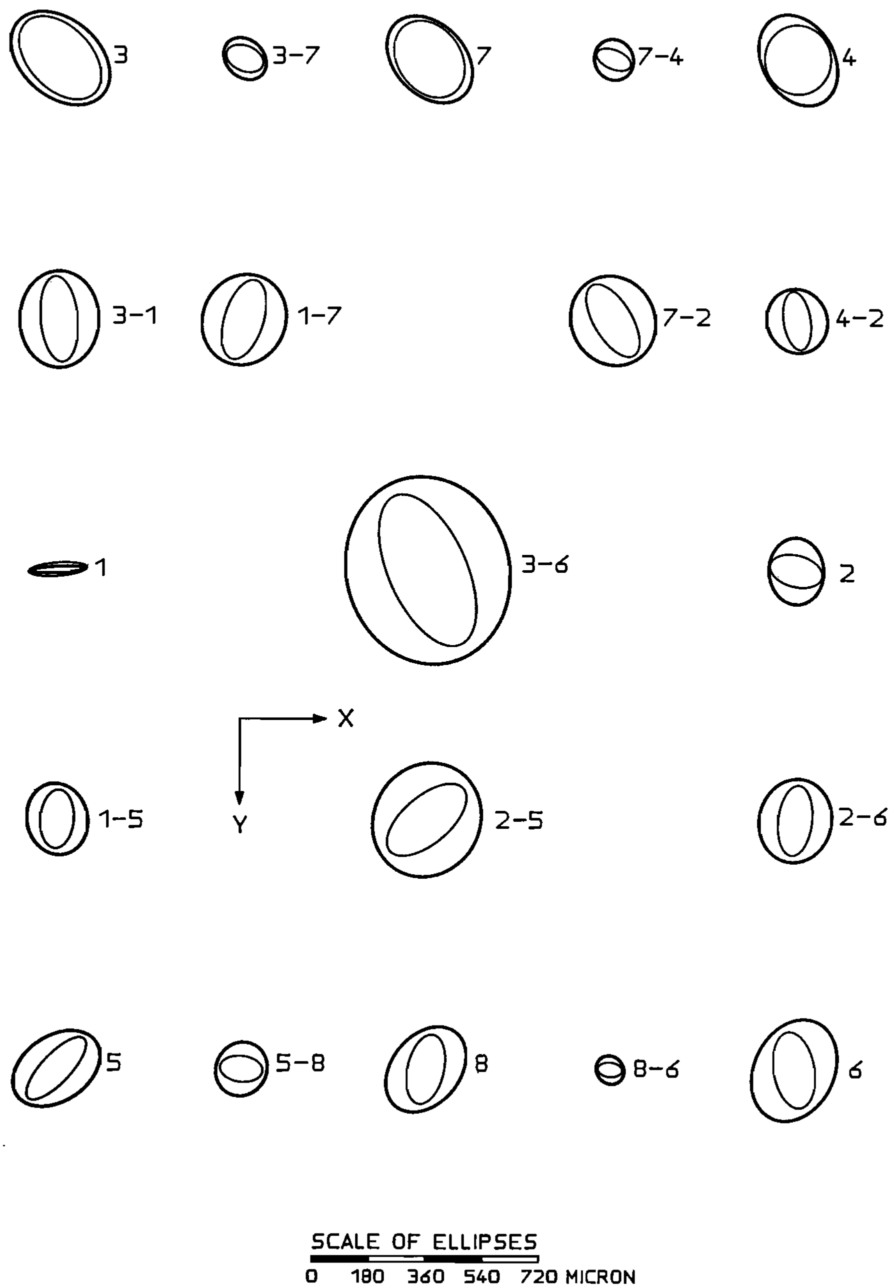


Fig. 3.13. Standard ellipses and relative standard ellipses in the $xy$-plane of experiment V.

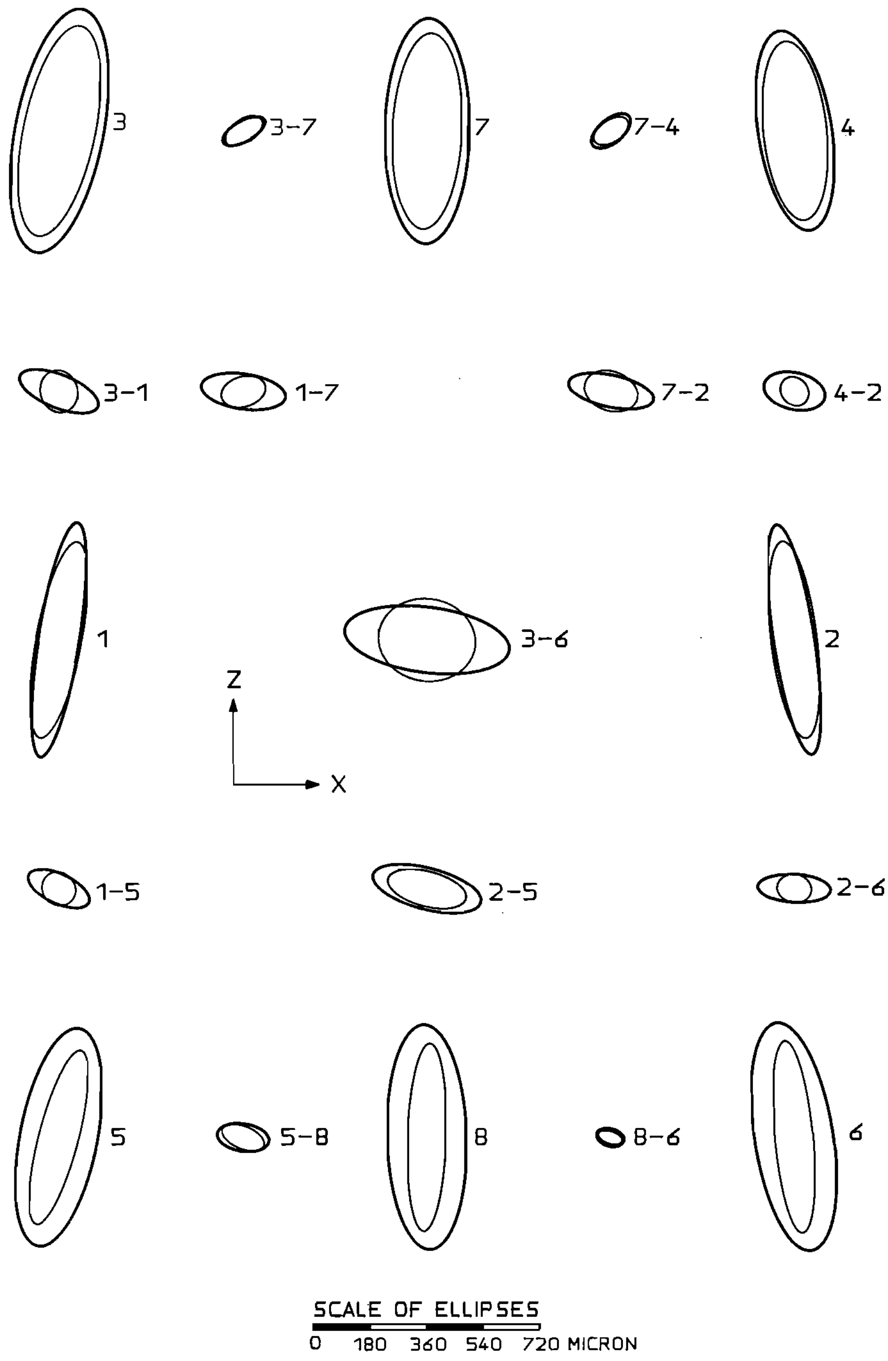


Fig. 3.14. Standard ellipses and relative standard ellipses in the *xz*-plane of experiment V.

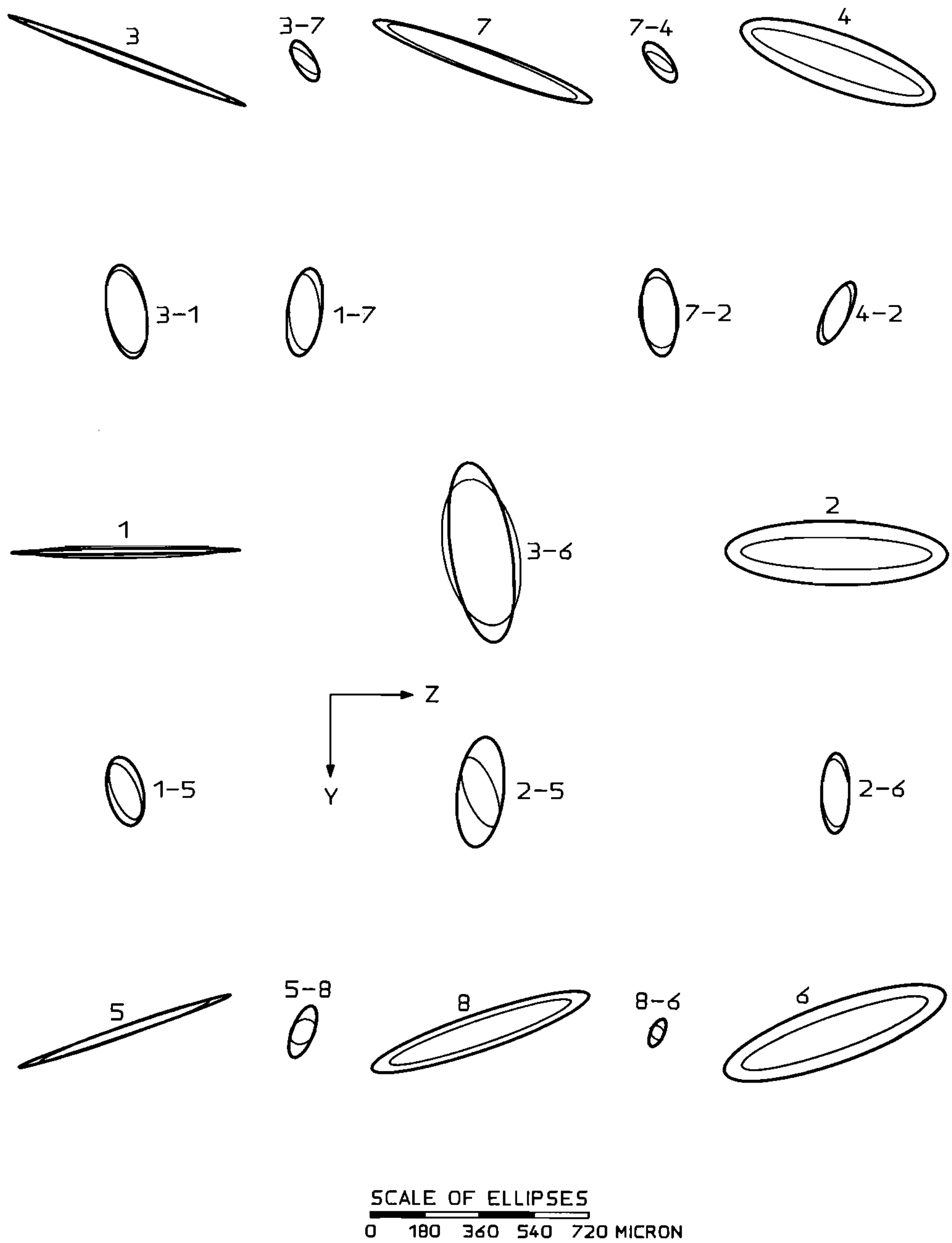


Fig. 3.15. Standard ellipses and relative standard ellipses in the *yz*-plane of experiment V.

# IV ERRORS DUE TO THE INNER ORIENTATION

## 1 General description

The third group of observation errors which influence the accuracy of the machine coordinates are the errors in inner orientation. The problem of inner orientation has three variables for each camera; the translations of the projection centre in three mutually perpendicular directions, the elements of inner orientation:

$$\left.\begin{array}{l}\text{left camera}\;\;: \Delta x',\ \Delta y' \text{ and } \Delta c' \\ \text{right camera}: \Delta x'',\ \Delta y'' \text{ and } \Delta c''\end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (4.1)$$

In appendix 3 the differential formulae are derived which give the relation between the differentials of the machine coordinates of a model point, $\Delta x$, $\Delta y$ and $\Delta h$, and the elements of inner orientation, as given in (4.1).

Pricked points and signalized points have been distinguished and the formulae are also here fitting to measurements with a Wild A7 and A8, in order to be able to compare the practical experiments with the mathematical description.

The formulae of appendix 3 can be summarized as follows:

$$\begin{pmatrix}\Delta x_I \\ \Delta y_I \\ \Delta y_{IL} \\ \Delta y_{IR} \\ \Delta h_I\end{pmatrix} = (A_I^i) \begin{pmatrix}\Delta x' \\ \Delta x'' \\ \Delta y' \\ \Delta y'' \\ \Delta c' \\ \Delta c''\end{pmatrix} \quad \ldots\ldots\ldots\ldots \quad (4.2)$$

The elements of matrix $(A_I^i)$ are functions of:

$x, y, z$: the coordinates of the concerning model point with the origin in the middle of the base; see figure 3.1.

$b$: the instrumental base

$z_0$: the mean $z$ of the model

$c$: the principal distance

In formula (4.2) three cases are to be distinguished for the $y$-coordinate:

- signalized points
- pricked points on left photo
- pricked points on right photo

The differentials of the $x$- and $h$-coordinate are the same for the different cases.

With simpler denotations:

$$\begin{pmatrix} \Delta x_I \\ \Delta y_I \\ \Delta y_{IL} \\ \Delta y_{IR} \\ \Delta h_I \end{pmatrix} \equiv (\Delta x_I^i) \qquad \begin{pmatrix} \Delta x' \\ \Delta x'' \\ \Delta y' \\ \Delta y'' \\ \Delta c' \\ \Delta c'' \end{pmatrix} \equiv (\Delta I) \quad \dots \quad (4.3)$$

(4.2) can be written:

$$(\Delta x_I^i) = (A_I^i)(\Delta I) \quad \dots \quad (4.4)$$

Application of the law of the propagation of errors to (4.4) gives:

$$\left(\sigma_{x_I^i x_I^j}\right) = (A_I^i)(\sigma_I)^2(A_I^j)^T \quad \dots \quad (4.5)$$

$(\sigma_I^2)$, the covariance matrix of the observations is under the assumption that there is no correlation:

$$(\sigma_I)^2 = \begin{pmatrix} \sigma_{x'}^2 & & & & & 0 \\ & \sigma_{x''}^2 & & & & \\ & & \sigma_{y'}^2 & & & \\ & & & \sigma_{y''}^2 & & \\ & & & & \sigma_{c'}^2 & \\ 0 & & & & & \sigma_{c''}^2 \end{pmatrix} \quad \dots \quad (4.6)$$

From the investigation made by ZORN 1954–1955 [4] the following standard deviations of the inner orientation were determined for fiducial marks of Wild camera's and plate carriers of Wild instruments:

$$\left.\begin{aligned} \sigma_{x'} = \sigma_{x''} = \sigma_{y'} = \sigma_{y''} &= 0.020 \text{ mm} \\ \sigma_{c'} = \sigma_{c''} &= 0.003 \text{ mm} \end{aligned}\right\} \quad \dots \quad (4.7)$$

The covariance matrix of the machine coordinates (4.5) can be computed with (4.7).

## 2 The experiments

Three experiments were executed, experiments VI, VII and VIII, in order to evaluate the covariance matrix $\left(\sigma_{x^i x^j}\right)$ of (4.5).

*Experiment VI*
Stereopair: 1887–1888
Photo scale: 1:15000
Camera: Wild RC5, $c = 152.47$ mm
Size: 23 × 23 cm
Distance model – projection centres: $z \simeq 430$ mm
Instrumental base: $b = 250$ mm

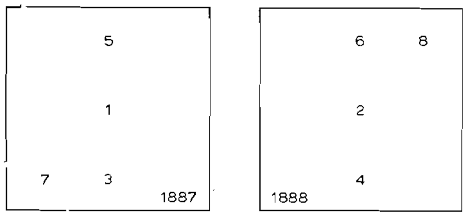


Fig. 4.1. The position of the 8 pricked points on the photographs.

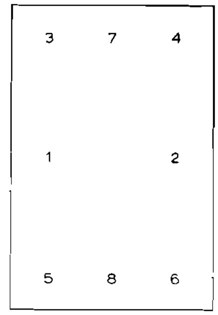


Fig. 4.2. The position of the points in the model.

Both the inner orientation and the relative orientation of this stereopair in the Wild A7 were repeated 20 times. The relative orientation is done numerically by measuring the parallaxes in six points. After each orientation the machine coordinates of 8 pricked points are measured in forward and backward sequence. Figure 4.1 gives the position of the 8 pricked points on the two photographs and figure 4.2 gives the position in the model. Points 1, 3, 5 and 7 are pricked on the left photo 1887 and points 2, 4, 6 and 8 on photo 1888.

Similar to the previous experiments an estimate for the covariance matrix $\left(\hat{\sigma}_{x^i x^j}\right)$ is computed according to formulae (3.10) to (3.12). The elements of this matrix are given in table 4.1 in square microns.

The covariance matrix of these machine coordinates can be computed by addition of the inner orientation (4.5), the relative orientation (3.8), and the measuring of a model point (2.4). The latter has to be multiplied by $\frac{1}{2}$ because of the measurements being made in forward and backward sequence. Simple addition can be applied indeed as the three groups of observations referring to the inner orientation, the relative orientation and the measuring of a point are mutually correlation free.

$$\left(\sigma_{x^i x^j}\right) = \left(\sigma_{x_I^i x_I^j}\right) + \left(\sigma_{x_O^i x_O^j}\right) + \tfrac{1}{2}\left(\sigma_{x_M^i x_M^j}\right) \quad \ldots\ldots\ldots\ldots \quad (4.8)$$

It follows from (4.5), (3.8) and (2.4) that:

$$\left(\sigma_{x^i x^j}\right) = (A_I^i)(\sigma_I^2)(A_I^j)^T + (A_O^i)(\sigma_{OO})(A_O^j)^T + \tfrac{1}{2}(A_M^i)(\sigma_M^2)(A_M^j)^T \quad \ldots\ldots \quad (4.9)$$

This covariance matrix $\left(\sigma_{x^i x^j}\right)$ is given in table 4.2 in square microns.

Sub-matrices of

$$\left(\hat{\sigma}_{x^i x^j}\right) \quad \text{and} \quad \left(\sigma_{x^i x^j}\right)$$

represent point standard ellipsoids and relative ellipsoids. As in the preceding chapters, the shape and position of the ellipsoids are represented by projections, ellipses. The elements of these ellipses are given in tables 4.3 and 4.4. The graphical presentation is in figures 4.3, 4.4 and 4.5; thin lines refer to $\left(\hat{\sigma}_{x^i x^j}\right)$ and thick lines refer to $\left(\sigma_{x^i x^j}\right)$.

Table 4.1. The elements of the estimated covariance matrix $\left(\hat{\sigma}_{x^i x^j}\right)$ of experiment VI.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ | $h_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 3205 | | | | | | | | | | | |
| $y_1$ | −551 | 2484 | | | | | | | | | | |
| $h_1$ | 6000 | 16 | 44341 | | | | | | | | | |
| $x_2$ | 783 | −642 | −12196 | 6073 | | | | | | | | |
| $y_2$ | −394 | 2840 | 673 | −669 | 4156 | | | | | | | |
| $h_2$ | 6029 | 63 | 43716 | −12103 | 831 | 43767 | | | | | | |
| $x_3$ | 4055 | 207 | 10250 | 87 | 1236 | 10036 | 6962 | | | | | |
| $y_3$ | 3727 | 3019 | 29142 | −8061 | 4354 | 28737 | 8135 | 24530 | | | | |
| $h_3$ | 6050 | 1040 | 41336 | −10737 | 2401 | 40718 | 11066 | 30184 | 40990 | | | |
| $x_4$ | 1274 | −407 | −8914 | 5215 | −15 | −8801 | 1752 | −5698 | −7641 | 5609 | | |
| $y_4$ | 3111 | 3383 | 26366 | −7675 | 5357 | 26243 | 7711 | 22363 | 27055 | −5184 | 22191 | |
| $h_4$ | 5695 | 1011 | 40102 | −10607 | 2245 | 39490 | 10490 | 28619 | 38943 | −7597 | 26228 | 37574 |
| $x_5$ | 3869 | −873 | 10398 | −508 | −1317 | 10550 | 4032 | 5902 | 9647 | −447 | 4577 | 9229 |
| $y_5$ | −3301 | 2570 | −23800 | 6707 | 2873 | −23752 | −4401 | −11642 | −20183 | 4922 | −10177 | −19856 |
| $h_5$ | 5157 | −385 | 45364 | −13970 | −398 | 45393 | 8321 | 27919 | 40499 | −10327 | 25308 | 39690 |
| $x_6$ | 374 | −780 | −15854 | 7125 | −1209 | −15756 | −1408 | −10612 | −14135 | 5251 | −10357 | −13953 |
| $y_6$ | −4414 | 2896 | −28025 | 7479 | 4268 | −28010 | −4612 | −13812 | −23750 | 6175 | −10857 | −23040 |
| $h_6$ | 6073 | −261 | 47581 | −14039 | −352 | 47787 | 9376 | 29649 | 42822 | −10450 | 26717 | 41793 |
| $x_7$ | 3010 | −2 | 3608 | 1784 | 841 | 3463 | 5110 | 3254 | 4515 | 3101 | 3230 | 4154 |
| $y_7$ | 3690 | 3421 | 27211 | −7335 | 5074 | 26657 | 8463 | 23682 | 28619 | −4879 | 22067 | 27198 |
| $h_7$ | 5699 | 1079 | 38820 | −10110 | 2625 | 38118 | 10672 | 28385 | 38490 | −7206 | 25906 | 36756 |
| $x_8$ | 2205 | −836 | −2292 | 3215 | −1217 | −2178 | 1510 | −1947 | −1684 | 2323 | −2538 | −1893 |
| $y_8$ | −3950 | 3039 | −26624 | 7346 | 4327 | −26526 | −4262 | −12565 | −22136 | 5887 | −9875 | −21674 |
| $h_8$ | 5512 | −474 | 45420 | −13692 | −578 | 45553 | 8446 | 27920 | 40601 | −10411 | 25163 | 39745 |

Table 4.2. The elements of the covariance matrix $\left(\sigma_{x^i x^j}\right)$ of the points used in experiment VI.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ | $h_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 2500.3 | | | | | | | | | | | |
| $y_1$ | 23.7 | 3330.6 | | | | | | | | | | |
| $h_1$ | 2144.7 | 374.5 | 33990.5 | | | | | | | | | |
| $x_2$ | 1344.2 | −133.9 | −12129.6 | 6551.6 | | | | | | | | |
| $y_2$ | −29.5 | 3164.9 | −453.2 | 164.0 | 3765.9 | | | | | | | |
| $h_2$ | 2351.9 | 351.8 | 31939.0 | −11435.3 | −438.0 | 31218.7 | | | | | | |
| $x_3$ | 2516.1 | 47.6 | 4312.4 | 527.0 | −60.9 | 4371.6 | 2814.6 | | | | | |
| $y_3$ | 1403.2 | 3337.3 | 22800.8 | −8309.7 | 2849.5 | 21636.2 | 2788.7 | 20201.4 | | | | |
| $h_3$ | 2013.6 | 361.2 | 32789.8 | −11924.8 | −354.9 | 31118.4 | 4001.8 | 24424.2 | 35119.4 | | | |
| $x_4$ | 1437.8 | −113.4 | −10267.7 | 5748.7 | 287.4 | −9625.1 | 765.3 | −7644.2 | −10969.8 | 5600.7 | | |
| $y_4$ | 1403.4 | 3400.5 | 20880.0 | −7683.8 | 2810.7 | 20161.4 | 2709.8 | 17742.5 | 21016.5 | −6798.5 | 17419.7 | |
| $h_4$ | 2280.0 | 339.3 | 30802.5 | −10989.1 | −708.4 | 29554.5 | 4176.9 | 22047.4 | 31707.6 | −9982.2 | 20384.5 | 30464.1 |
| $x_5$ | 2684.5 | 62.3 | 5644.5 | 121.8 | −76.7 | 5647.0 | 2993.4 | 3750.3 | 5381.9 | 410.4 | 3558.0 | 5451.7 |
| $y_5$ | −1119.6 | 3001.2 | −18385.8 | 6751.1 | 3505.9 | −17515.6 | −2320.1 | −8657.3 | −16915.4 | 5353.0 | −7976.5 | −16155.9 |
| $h_5$ | 2157.3 | 391.1 | 35499.8 | −13008.7 | −558.8 | 33822.8 | 4470.4 | 22714.8 | 32666.5 | −10314.8 | 21565.8 | 31201.9 |
| $x_6$ | 1314.6 | −145.1 | −13142.8 | 6907.5 | −13.4 | −12457.0 | 406.8 | −8405.2 | −12061.9 | 5691.1 | −7969.1 | −11122.3 |
| $y_6$ | −1371.4 | 2934.2 | −21351.4 | 7936.4 | 3404.9 | −20646.1 | −2740.5 | −11135.1 | −20424.1 | 6441.7 | −10241.0 | −19344.6 |
| $h_6$ | 2434.5 | 365.3 | 33158.4 | −11930.9 | −108.5 | 31959.5 | 4576.2 | 21410.5 | 70395.7 | −9280.8 | 20301.3 | 29264.2 |
| $x_7$ | 2123.3 | −14.5 | −1316.3 | 2549.0 | 31.1 | −1000.4 | 2006.5 | −1310.2 | −1880.4 | 2625.1 | −935.9 | −1302.0 |
| $y_7$ | 1511.0 | 3335.8 | 21658.5 | −7772.7 | 2771.5 | 20656.4 | 2837.6 | 18851.5 | 22712.1 | −7146.3 | 17118.3 | 21061.4 |
| $h_7$ | 2185.2 | 345.8 | 31393.3 | −11241.2 | −486.5 | 29943.8 | 4103.7 | 22889.8 | 32917.0 | −10335.3 | 20261.6 | 30528.5 |
| $x_8$ | 2036.8 | −37.9 | −3430.5 | 3392.8 | 16.5 | −3060.0 | 1754.7 | −2050.8 | −2942.9 | 2957.4 | −1970.7 | −2519.3 |
| $y_8$ | −1446.1 | 2955.6 | −19409.7 | 7024.8 | 3437.8 | −18786.0 | −2708.0 | −9328.6 | −17831.4 | 5446.0 | −8740.9 | −17164.0 |
| $h_8$ | 2401.3 | 366.8 | 33298.4 | −11949.2 | −338.2 | 31957.0 | 4564.6 | 21241.1 | 30552.1 | −9298.0 | 20242.8 | 29260.1 |

| $h_4$ | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 37574 | | | | | | | | | | | | |
| 9229 | 5924 | | | | | | | | | | | |
| −19856 | − 6163 | 16433 | | | | | | | | | | |
| 39690 | 10252 | −26488 | 49879 | | | | | | | | | |
| −13953 | − 630 | 8775 | −17963 | 9215 | | | | | | | | |
| −23040 | − 8858 | 19075 | −31204 | 9186 | 24473 | | | | | | | |
| 41793 | 11652 | −27277 | 51537 | −17994 | −32949 | 54037 | | | | | | |
| 4154 | 2327 | − 1237 | 1973 | 738 | − 886 | 2552 | 4437 | | | | | |
| 27198 | 5271 | −10007 | 25288 | − 9850 | −11393 | 26931 | 3747 | 23518 | | | | |
| 36756 | 8868 | −18753 | 37629 | −13354 | −21592 | 39781 | 4380 | 27292 | 36567 | | | |
| − 1893 | 2736 | 1130 | − 3573 | 4129 | − 95 | − 2787 | 1637 | − 1885 | − 1689 | 3449 | | |
| −21674 | − 7966 | 18551 | −29850 | 9113 | 23200 | −31255 | − 741 | −10422 | −20292 | 368 | 22427 | |
| 39745 | 10901 | −26414 | 49529 | −17416 | −31640 | 51619 | 1946 | 25288 | 37840 | −2910 | −30245 | 49680 |

| $h_4$ | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 30464.1 | | | | | | | | | | | | |
| 5451.7 | 3370.3 | | | | | | | | | | | |
| −16155.9 | −3022.5 | 14159.6 | | | | | | | | | | |
| 31201.9 | 5824.1 | −20696.4 | 39952.1 | | | | | | | | | |
| −11122.3 | − 15.0 | 7679.0 | −14796.0 | 7889.6 | | | | | | | | |
| −19344.6 | −3577.5 | 15133.1 | −22963.9 | 8896.3 | 17961.1 | | | | | | | |
| 29264.2 | 5860.5 | −18840.1 | 36376.4 | −13773.6 | −22288.0 | 35176.3 | | | | | | |
| − 1302.0 | 2020.4 | 600.8 | − 1157.8 | 2444.8 | 777.1 | − 727.5 | 2373.4 | | | | | |
| 21061.4 | 3743.5 | − 8092.2 | 21647.1 | − 7840.0 | −10425.4 | 20403.6 | −1131.4 | 18438.6 | | | | |
| 30528.5 | 5413.9 | −16246.3 | 31376.8 | −11338.5 | −19573.1 | 29579.5 | −1636.5 | 22033.5 | 31935.0 | | | |
| − 2519.3 | 1712.7 | 2199.4 | − 4237.9 | 3753.9 | 2372.1 | − 3570.8 | 2223.1 | − 1799.2 | − 2601.9 | 2838.6 | | |
| −17164.0 | −3457.0 | 14339.8 | −21434.8 | 8116.6 | 16276.0 | −20385.0 | 386.4 | − 8787.8 | −17204.3 | 2018.1 | 15710.9 | |
| 29260.1 | 5843.2 | −19111.0 | 36897.7 | −13769.0 | −21786.8 | 34612.4 | − 713.7 | 20296.9 | 29424.5 | −3507.1 | −20689.8 | 35390.8 |

Table 4.3 The elements of the ellipses computed from $\left(\hat{\sigma}_{x^i x^j}\right)$ pertaining to experiment VI.

| | | z = 0 | | | y = 0 | | | x = 0 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | a | b | ψ | a | b | ψ | a | b | ψ |
| | 1 | 59 | 47 | 132 | 213 | 48 | 9 | 211 | 50 | 0 |
| | 2 | 79 | 63 | 119 | 218 | 50 | 182 | 209 | 64 | 1 |
| | 3 | 166 | 61 | 24 | 210 | 61 | 18 | 253 | 38 | 42 |
| | 4 | 154 | 64 | 182 | 198 | 62 | 186 | 239 | 50 | 41 |
| | 5 | 139 | 55 | 172 | 228 | 60 | 14 | 254 | 43 | 168 |
| | 6 | 170 | 70 | 28 | 246 | 54 | 178 | 275 | 56 | 163 |
| | 7 | 156 | 61 | 12 | 193 | 62 | 8 | 241 | 45 | 43 |
| | 8 | 150 | 59 | 1 | 223 | 57 | 196 | 263 | 54 | 163 |
| 1 | 2 | 88 | 31 | 102 | 88 | 26 | 99 | 32 | 25 | 79 |
| 1 | 3 | 147 | 37 | 12 | 56 | 39 | 38 | 145 | 52 | 100 |
| 1 | 4 | 149 | 43 | 169 | 82 | 35 | 80 | 134 | 40 | 104 |
| 1 | 5 | 119 | 30 | 188 | 61 | 34 | 19 | 119 | 55 | 113 |
| 1 | 6 | 176 | 42 | 39 | 111 | 52 | 115 | 149 | 46 | 115 |
| 1 | 7 | 138 | 40 | 198 | 62 | 33 | 29 | 139 | 57 | 104 |
| 1 | 8 | 142 | 32 | 16 | 57 | 47 | 191 | 139 | 51 | 112 |
| 2 | 3 | 176 | 41 | 42 | 113 | 58 | 102 | 141 | 58 | 100 |
| 2 | 4 | 126 | 32 | 8 | 49 | 34 | 185 | 126 | 47 | 107 |
| 2 | 5 | 162 | 41 | 152 | 115 | 51 | 90 | 123 | 52 | 108 |
| 2 | 6 | 143 | 28 | 7 | 48 | 31 | 185 | 144 | 38 | 113 |
| 2 | 7 | 152 | 36 | 34 | 85 | 61 | 121 | 133 | 63 | 105 |
| 2 | 8 | 143 | 26 | 177 | 60 | 42 | 63 | 135 | 45 | 109 |
| 3 | 4 | 96 | 43 | 92 | 95 | 25 | 95 | 48 | 19 | 73 |
| 3 | 5 | 253 | 69 | 1 | 107 | 56 | 29 | 254 | 98 | 105 |
| 3 | 6 | 302 | 66 | 27 | 139 | 94 | 113 | 279 | 91 | 108 |
| 3 | 7 | 35 | 25 | 81 | 36 | 22 | 77 | 33 | 12 | 54 |
| 3 | 8 | 274 | 65 | 14 | 101 | 82 | 30 | 269 | 94 | 106 |
| 4 | 5 | 259 | 67 | 177 | 121 | 77 | 67 | 244 | 86 | 107 |
| 4 | 6 | 263 | 57 | 8 | 92 | 63 | 182 | 265 | 79 | 111 |
| 4 | 7 | 62 | 39 | 94 | 62 | 25 | 103 | 42 | 22 | 77 |
| 4 | 8 | 256 | 58 | 192 | 92 | 60 | 26 | 256 | 82 | 108 |
| 5 | 6 | 131 | 46 | 86 | 129 | 27 | 106 | 56 | 21 | 125 |
| 5 | 7 | 246 | 70 | 193 | 115 | 61 | 30 | 246 | 103 | 107 |
| 5 | 8 | 66 | 37 | 75 | 62 | 22 | 100 | 44 | 19 | 120 |
| 6 | 7 | 281 | 62 | 22 | 120 | 93 | 144 | 270 | 95 | 111 |
| 6 | 8 | 67 | 21 | 91 | 67 | 19 | 111 | 28 | 14 | 149 |
| 7 | 8 | 260 | 62 | 7 | 104 | 66 | 12 | 261 | 97 | 109 |

Table 4.4. The elements of the ellipses computed from $\left(\sigma_{x^i x^j}\right)$ pertaining to the points used in experiment VI.

| | | $z = 0$ | | | $y = 0$ | | | $x = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| | 1 | 58 | 50 | 2 | 185 | 49 | 4 | 184 | 58 | 1 |
| | 2 | 81 | 61 | 96 | 189 | 45 | 176 | 177 | 61 | 199 |
| | 3 | 144 | 49 | 10 | 189 | 48 | 8 | 231 | 46 | 41 |
| | 4 | 143 | 50 | 173 | 184 | 46 | 178 | 213 | 50 | 40 |
| | 5 | 122 | 51 | 184 | 202 | 50 | 10 | 227 | 52 | 168 |
| | 6 | 152 | 52 | 34 | 202 | 46 | 175 | 225 | 52 | 162 |
| | 7 | 136 | 48 | 196 | 179 | 48 | 196 | 220 | 46 | 41 |
| | 8 | 127 | 50 | 10 | 189 | 50 | 193 | 220 | 51 | 164 |
| 1 | 2 | 80 | 27 | 96 | 80 | 36 | 94 | 37 | 28 | 4 |
| 1 | 3 | 130 | 13 | 5 | 59 | 17 | 196 | 131 | 58 | 92 |
| 1 | 4 | 137 | 20 | 166 | 72 | 53 | 96 | 118 | 53 | 103 |
| 1 | 5 | 109 | 13 | 189 | 54 | 22 | 4 | 110 | 48 | 116 |
| 1 | 6 | 151 | 22 | 39 | 89 | 52 | 111 | 124 | 53 | 105 |
| 1 | 7 | 125 | 13 | 189 | 56 | 24 | 191 | 123 | 56 | 98 |
| 1 | 8 | 119 | 17 | 17 | 53 | 35 | 187 | 115 | 51 | 108 |
| 2 | 3 | 160 | 30 | 37 | 91 | 64 | 98 | 137 | 60 | 88 |
| 2 | 4 | 125 | 25 | 3 | 54 | 19 | 178 | 125 | 51 | 98 |
| 2 | 5 | 141 | 27 | 152 | 101 | 55 | 84 | 110 | 49 | 122 |
| 2 | 6 | 123 | 23 | 5 | 53 | 18 | 177 | 123 | 47 | 110 |
| 2 | 7 | 140 | 30 | 26 | 67 | 52 | 140 | 130 | 56 | 93 |
| 2 | 8 | 121 | 25 | 175 | 52 | 51 | 31 | 114 | 48 | 112 |
| 3 | 4 | 84 | 44 | 88 | 84 | 45 | 89 | 62 | 20 | 50 |
| 3 | 5 | 227 | 12 | 198 | 99 | 14 | 200 | 228 | 98 | 103 |
| 3 | 6 | 263 | 33 | 23 | 108 | 83 | 142 | 246 | 93 | 99 |
| 3 | 7 | 35 | 30 | 74 | 37 | 32 | 45 | 44 | 15 | 45 |
| 3 | 8 | 237 | 21 | 11 | 98 | 44 | 190 | 234 | 97 | 100 |
| 4 | 5 | 235 | 26 | 176 | 94 | 86 | 53 | 220 | 85 | 109 |
| 4 | 6 | 237 | 43 | 4 | 94 | 21 | 170 | 237 | 83 | 104 |
| 4 | 7 | 52 | 40 | 91 | 52 | 37 | 99 | 51 | 19 | 54 |
| 4 | 8 | 228 | 34 | 189 | 89 | 45 | 181 | 226 | 84 | 105 |
| 5 | 6 | 108 | 39 | 89 | 107 | 48 | 93 | 58 | 30 | 157 |
| 5 | 7 | 224 | 15 | 189 | 96 | 41 | 199 | 222 | 93 | 106 |
| 5 | 8 | 53 | 34 | 90 | 56 | 35 | 73 | 47 | 23 | 157 |
| 6 | 7 | 247 | 38 | 17 | 101 | 55 | 162 | 239 | 89 | 101 |
| 6 | 8 | 57 | 32 | 88 | 57 | 37 | 98 | 45 | 20 | 154 |
| 7 | 8 | 228 | 26 | 3 | 94 | 19 | 186 | 228 | 92 | 103 |

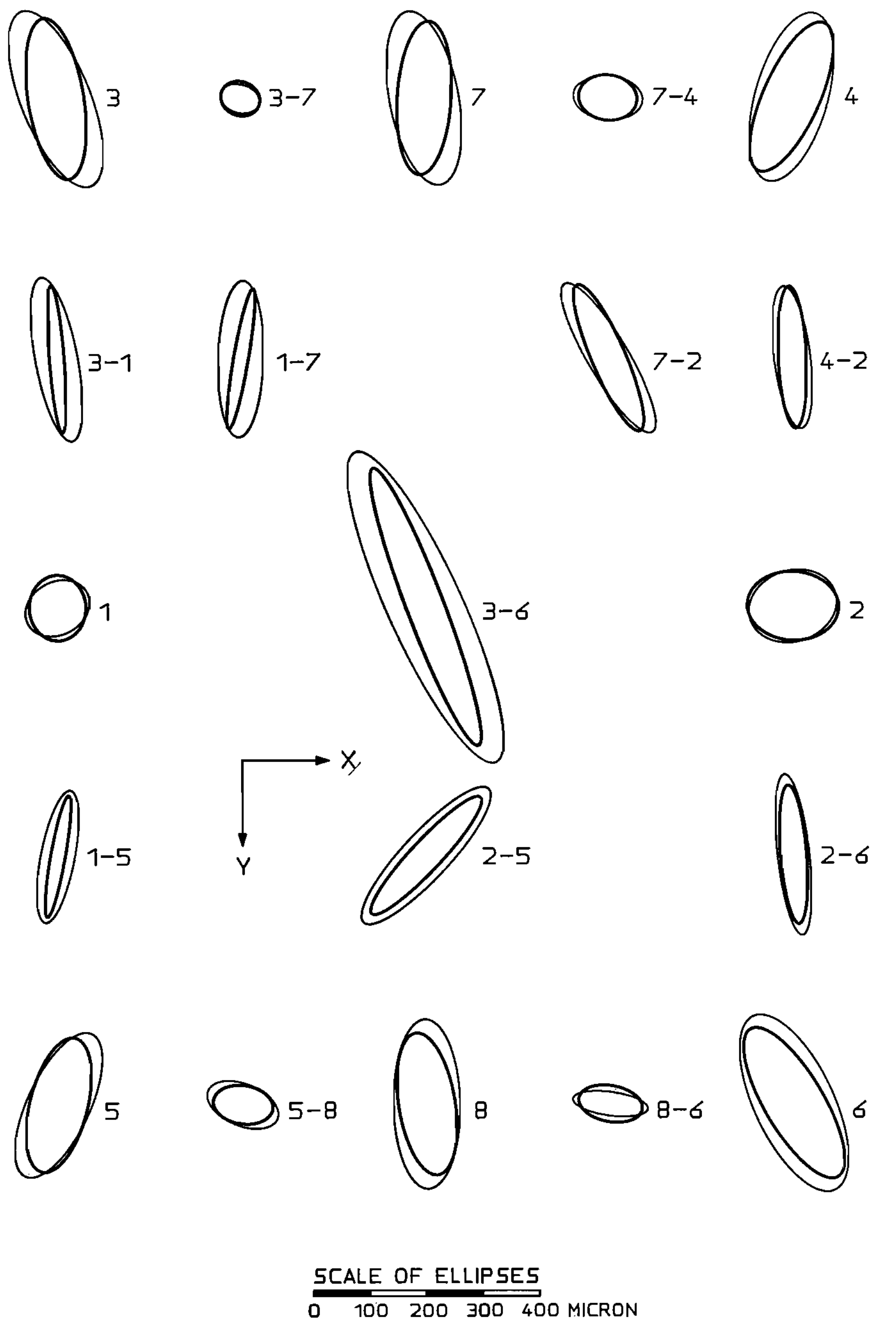


Fig. 4.3. Standard ellipses and relative standard ellipses in the $xy$-plane of experiment VI.

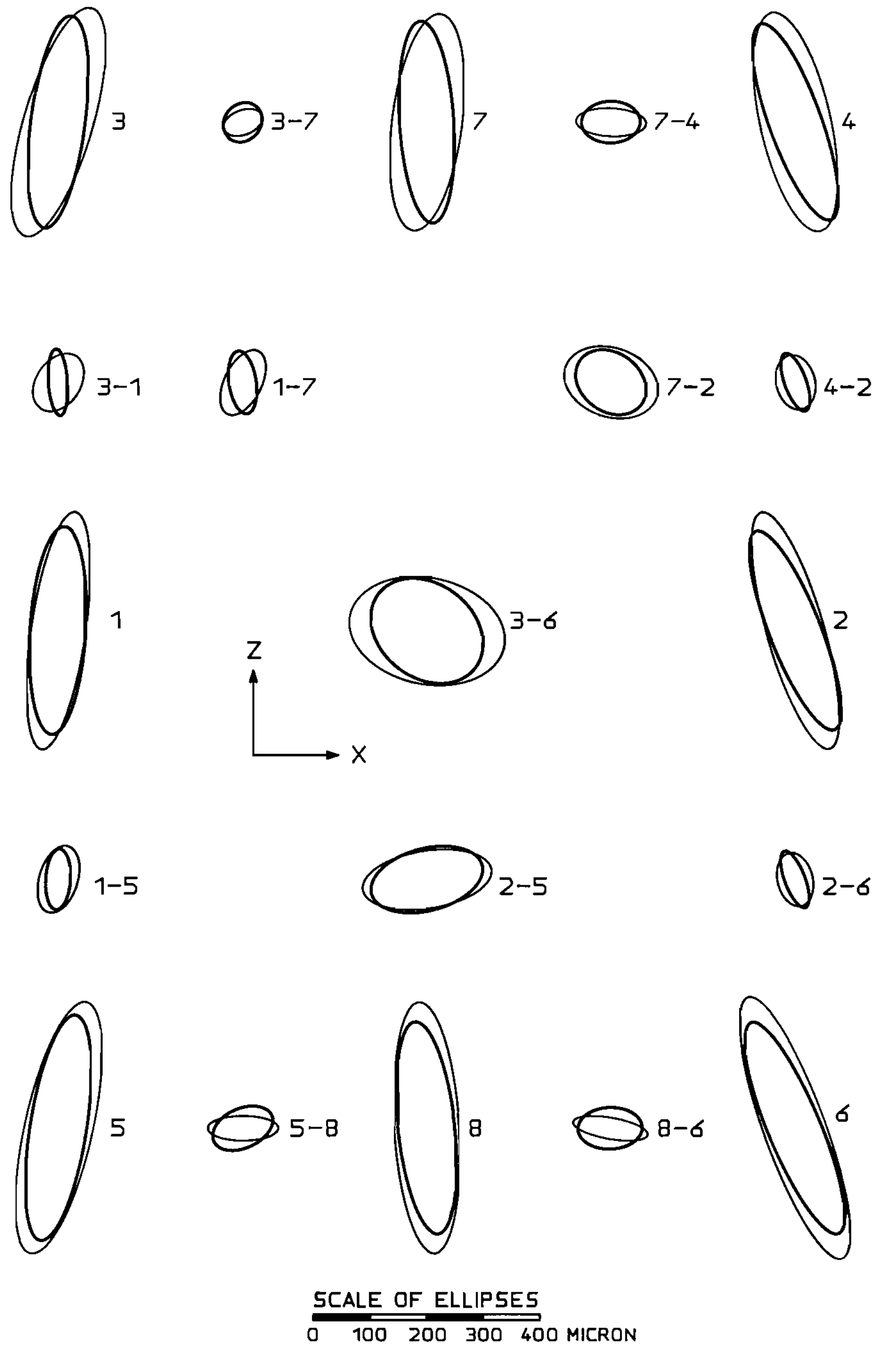


Fig. 4.4. Standard ellipses and relative standard ellipses in the $xz$-plane of experiment VI.

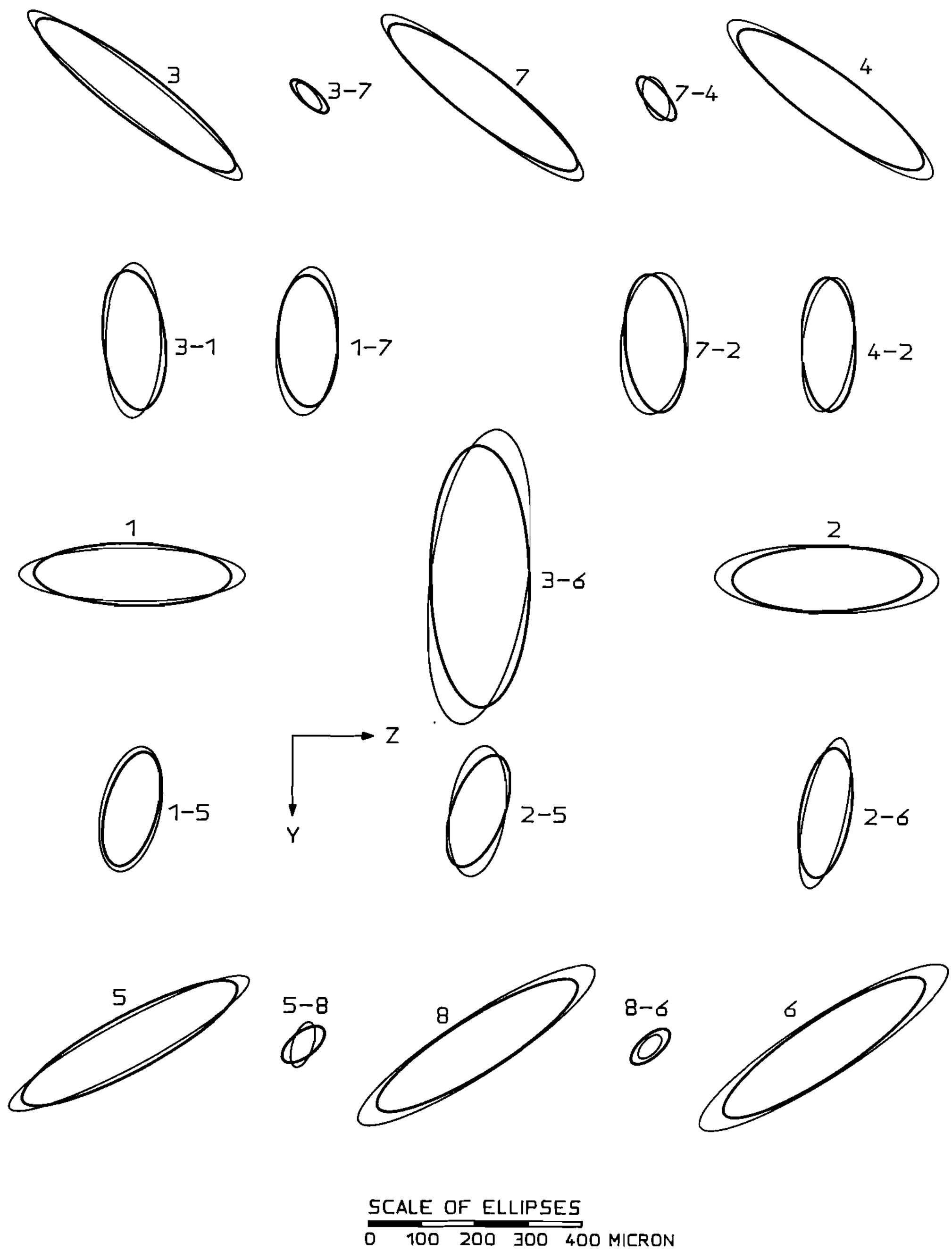


Fig. 4.5. Standard ellipses and relative standard ellipses in the $yz$-plane of experiment VI.

*Experiment VII*
Stereopair: 147–149
Photo scale: 1:5000
Camera: Wild RC 5, $c$ = 152.15 mm
Size: 23 × 23 cm
Distance model – projection centres: $z \simeq 290$ mm
Instrumental base: $b$ = 200 mm

This experiment is similar to the previous one, with two differences, firstly a different stereopair and secondly the Wild A8 is used instead of the Wild A7. After each inner orientation and empirical relative orientation, the coordinates of 8 pricked points are measured in forward and backward sequence. The position of the 8 points is given in figures 4.6 and 4.7.

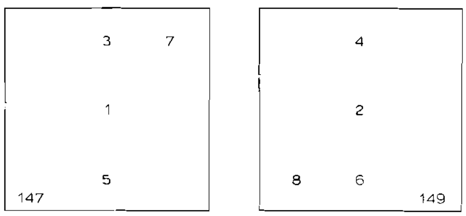


Fig. 4.6. The position of the 8 pricked points on the photograph.

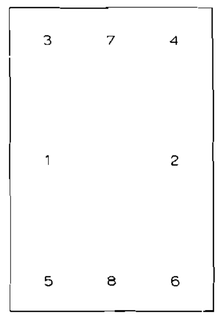


Fig. 4.7. The position of the points in the model.

The elements of the estimated covariance matrix $\left(\hat{\sigma}_{x^i x^j}\right)$ are computed from the 20 observations of each model point according to (3.12) and given in table 4.5.

The covariance matrix $\left(\sigma_{x^i x^j}\right)$, computed according to (4.9), is given in table 4.6.

The form of the ellipsoids, given by the covariance matrices, are again presented by ellipses, see figure 4.8, 4.9 and 4.10; thin lines refer to $\left(\hat{\sigma}_{x^i x^j}\right)$ and thick lines refer to $\left(\sigma_{x^i x^j}\right)$.

The elements of the ellipses are given in tables 4.7 and 4.8 respectively.

Table 4.5. The elements of the estimated covariance matrix $\left(\hat{\sigma}_{x^i x^j}\right)$ of experiment VII.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ | $h_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 8675 | | | | | | | | | | | |
| $y_1$ | 3376 | 8087 | | | | | | | | | | |
| $h_1$ | 7049 | 2809 | 30672 | | | | | | | | | |
| $x_2$ | 2999 | 1847 | −10967 | 8568 | | | | | | | | |
| $y_2$ | 1170 | 162 | 36 | 1353 | 3691 | | | | | | | |
| $h_2$ | 12259 | 4193 | 29593 | − 6761 | − 79 | 34783 | | | | | | |
| $x_3$ | 5797 | − 4266 | 990 | 3820 | 4713 | 4904 | 14451 | | | | | |
| $y_3$ | 6039 | 7495 | 17303 | − 4088 | 194 | 17064 | − 2651 | 15396 | | | | |
| $h_3$ | 6314 | 2645 | 26509 | − 9578 | −1292 | 25682 | − 352 | 16374 | 25125 | | | |
| $x_4$ | 2634 | − 5779 | − 8925 | 6868 | 5384 | − 5302 | 14459 | − 9387 | −10070 | 18510 | | |
| $y_4$ | 8335 | 3109 | 13701 | − 3405 | 3197 | 20989 | 6550 | 12155 | 16136 | 149 | 16563 | |
| $h_4$ | 9936 | 3488 | 26281 | − 6905 | − 574 | 29578 | 3223 | 16516 | 24748 | − 6771 | 18922 | 27604 |
| $x_5$ | 10604 | 10333 | 9794 | 3311 | −1986 | 16351 | − 2333 | 12266 | 9641 | − 7296 | 8154 | 13352 |
| $y_5$ | − 1234 | 7084 | −15513 | 8120 | − 371 | −13877 | − 6284 | − 1892 | −12436 | − 2281 | − 8290 | −11787 |
| $h_5$ | 8205 | 2818 | 33733 | −11708 | 415 | 33397 | 2004 | 17788 | 27823 | − 8204 | 20514 | 28138 |
| $x_6$ | 4493 | 8947 | − 9646 | 9116 | −2663 | − 4275 | − 5570 | 2351 | − 6282 | − 4586 | − 4643 | − 3683 |
| $y_6$ | − 6022 | − 1987 | −20316 | 6936 | 3718 | −23117 | 1933 | −10307 | −17985 | 9009 | −10160 | −19317 |
| $h_7$ | 12879 | 4321 | 33179 | − 8083 | 608 | 38760 | 5700 | 17691 | 27156 | − 4756 | 23081 | 31455 |
| $x_7$ | 5155 | − 4439 | − 1378 | 4701 | 5047 | 2603 | 14551 | − 4268 | − 2902 | 15625 | 5138 | 708 |
| $y_7$ | 5753 | 5946 | 16694 | − 4121 | 830 | 16435 | − 843 | 13930 | 15575 | − 7201 | 12317 | 15958 |
| $h_7$ | 7092 | 2579 | 25870 | − 8808 | −1382 | 26217 | 374 | 15882 | 24543 | − 9291 | 16321 | 25086 |
| $x_8$ | 8210 | 10478 | 0 | 6720 | −2628 | 6693 | − 4347 | 7865 | 1811 | − 6504 | 1860 | 5356 |
| $y_8$ | − 5102 | 2107 | −19492 | 7278 | 1421 | −21147 | − 3357 | − 6838 | −16180 | 2559 | −10663 | −17219 |
| $h_8$ | 11229 | 3646 | 33340 | − 9321 | 662 | 36282 | 4678 | 17860 | 27469 | − 5662 | 22020 | 29927 |

Table 4.6. The elements of the covariance matrix $\left(\sigma_{x^i x^j}\right)$ of the points used in experiment VII.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ | $h_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 2478.3 | | | | | | | | | | | |
| $y_1$ | 38.2 | 1412.3 | | | | | | | | | | |
| $h_1$ | 3186.7 | 328.8 | 13289.1 | | | | | | | | | |
| $x_2$ | 288.1 | − 98.7 | − 4385.1 | 2689.9 | | | | | | | | |
| $y_2$ | 113.9 | 187.7 | − 119.7 | − 10.8 | 3307.4 | | | | | | | |
| $h_2$ | 4012.9 | 316.4 | 12700.8 | −3601.7 | − 81.4 | 13420.2 | | | | | | |
| $x_3$ | 2151.1 | −1036.3 | 1160.2 | 899.4 | 3645.6 | 2081.9 | 7366.5 | | | | | |
| $y_3$ | 1999.4 | 1229.2 | 7921.6 | −2606.4 | 517.9 | 7704.8 | 470.1 | 6132.2 | | | | |
| $h_3$ | 3296.2 | 274.5 | 12752.4 | −4095.1 | − 891.6 | 12524.9 | 392.0 | 8134.6 | 13462.4 | | | |
| $x_4$ | 689.2 | −1219.2 | − 3884.4 | 2452.6 | 3714.0 | − 3009.2 | 6686.0 | −2792.9 | − 4763.4 | 8210.2 | | |
| $y_4$ | 2779.7 | 536.0 | 8378.0 | −2474.1 | 3686.3 | 8743.0 | 5641.2 | 6065.3 | 7603.1 | 2112.5 | 10681.3 | |
| $h_4$ | 3961.6 | 323.1 | 12756.4 | −3661.1 | − 162.3 | 13212.1 | 1997.7 | 8145.1 | 13152.7 | −3346.8 | 9130.9 | 13939.8 |
| $x_5$ | 2321.7 | 1011.0 | 3527.5 | 223.7 | −3039.3 | 4293.7 | −2690.4 | 2403.2 | 4305.0 | −4268.0 | − 790.3 | 4163.2 |
| $y_5$ | −1836.0 | 1147.9 | − 7212.3 | 2432.5 | − 441.2 | − 6993.7 | −2594.7 | −3185.6 | − 6511.7 | 12.5 | − 5005.9 | − 6715.3 |
| $h_5$ | 3179.8 | 379.6 | 13024.3 | −4401.5 | 895.2 | 12609.7 | 2358.2 | 7601.7 | 11735.1 | −2442.7 | 9323.1 | 12112.9 |
| $x_6$ | 208.4 | 952.7 | − 3918.0 | 2577.5 | −3415.8 | − 3182.8 | −4238.1 | −1865.2 | − 2567.9 | −3004.4 | − 6011.0 | − 2974.4 |
| $y_6$ | −2276.3 | 94.7 | − 7863.3 | 2215.9 | 3766.4 | − 8141.7 | 3050.1 | −3864.7 | − 8446.2 | 6062.8 | − 716.2 | − 7872.4 |
| $h_6$ | 3897.6 | 303.1 | 12424.2 | −3557.1 | − 29.2 | 12961.2 | 1998.4 | 7170.3 | 11728.7 | −2753.7 | 8240.8 | 12375.1 |
| $x_7$ | 1726.0 | −1087.3 | − 407.7 | 1399.9 | 3734.1 | 530.8 | 7240.9 | − 521.5 | − 1210.5 | 7254.7 | 4654.2 | 361.11 |
| $y_7$ | 2061.7 | 1000.8 | 7724.2 | −2530.9 | 1411.9 | 7577.0 | 1871.8 | 5851.5 | 7644.6 | −1370.4 | 7073.8 | 8039.4 |
| $h_7$ | 3446.6 | 283.6 | 12519.6 | −3892.6 | − 667.1 | 12503.6 | 885.5 | 7931.6 | 13032.7 | −4272.6 | 7886.2 | 13114.0 |
| $x_8$ | 1277.2 | 1073.2 | − 283.1 | 1482.6 | −3750.7 | 532.6 | −4170.3 | 231.1 | 962.2 | −4324.9 | − 4060.3 | 575.2 |
| $y_8$ | −2189.8 | 624.6 | − 7474.7 | 2241.1 | 1724.6 | − 7646.2 | 163.1 | −3413.6 | − 7346.9 | 2946.7 | − 2720.5 | − 7206.7 |
| $h_8$ | 3398.1 | 330.9 | 12124.5 | −3808.5 | 519.7 | 12187.8 | 2218.2 | 7014.5 | 11078.3 | −2329.9 | 8466.5 | 11595.0 |

| $h_4$ | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 27604 | | | | | | | | | | | | |
| 13352 | 21714 | | | | | | | | | | | |
| −11787 | 4973 | 17543 | | | | | | | | | | |
| 28138 | 11001 | −18014 | 38341 | | | | | | | | | |
| − 3683 | 14601 | 15807 | −11322 | 21061 | | | | | | | | |
| −19317 | −11638 | 10221 | −23186 | 2149 | 19914 | | | | | | | |
| 31455 | 16876 | −16439 | 38524 | − 6938 | −26080 | 44808 | | | | | | |
| 708 | − 3341 | − 5178 | − 299 | − 5243 | 3670 | 3511 | 14991 | | | | | |
| 15958 | 10053 | − 3333 | 17290 | 224 | − 9358 | 17161 | − 2390 | 12993 | | | | |
| 25086 | 10497 | −12280 | 27360 | − 5404 | −18374 | 27729 | − 2184 | 15250 | 24420 | | | |
| 5356 | 19801 | 11324 | − 243 | 19411 | − 5356 | 5553 | − 4744 | 5513 | 2798 | 21430 | | |
| −17219 | − 5005 | 14618 | −22723 | 8932 | 16691 | −24515 | − 1876 | − 7062 | −16398 | 1969 | 17425 | |
| 29927 | 14441 | −17177 | 38286 | − 8768 | −24452 | 41709 | 2384 | 17363 | 27503 | 3184 | −23800 | 40388 |

| $h_4$ | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13939.8 | | | | | | | | | | | | |
| 4163.2 | 6480.7 | | | | | | | | | | | |
| − 6715.3 | − 351.0 | 5788.0 | | | | | | | | | | |
| 12112.9 | 2553.6 | −7787.9 | 14091.4 | | | | | | | | | |
| − 2974.4 | 4630.5 | 4140.7 | − 5276.7 | 7473.1 | | | | | | | | |
| − 7872.4 | −6191.6 | 3885.3 | − 6770.8 | −1880.5 | 9746.7 | | | | | | | |
| 12375.1 | 4272.7 | −7113.4 | 12821.9 | −3355.7 | −8313.7 | 13322.2 | | | | | | |
| 361.11 | −3224.4 | −1802.1 | 903.1 | −3914.2 | 4057.7 | 551.3 | 7409.6 | | | | | |
| 8039.4 | 1289.8 | −3511.6 | 7700.1 | −3027.7 | −2703.5 | 7015.2 | 860.4 | 6199.3 | | | | |
| 13114.0 | 4203.3 | −6444.3 | 11616.5 | −2630.1 | −8129.8 | 11701.6 | − 781.1 | 7745.4 | 13006.0 | | | |
| 575.2 | 6204.5 | 2184.8 | − 1665.9 | 6818.9 | −4658.7 | 428.5 | −4282.6 | −1088.6 | 847.4 | 7404.7 | | |
| − 7206.7 | −3489.6 | 4790.5 | − 7269.3 | 1052.5 | 6978.5 | − 7890.7 | 1055.3 | −2991.3 | − 7182.4 | −1447.4 | 6289.3 | |
| 11595.0 | 3185.5 | −7124.1 | 12869.5 | −4280.3 | −7050.8 | 12463.2 | 839.1 | 7027.4 | 11027.0 | − 671.0 | −7369.0 | 12367.9 |

Table 4.7. The elements of the ellipses computed from $\left(\hat{\sigma}_{x^i x^j}\right)$ pertaining to experiment VII.

| | | $z = 0$ | | | $y = z$ | | | $x = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| | 1 | 108 | 71 | 53 | 181 | 81 | 18 | 176 | 88 | 8 |
| | 2 | 94 | 58 | 84 | 191 | 83 | 185 | 187 | 61 | 200 |
| | 3 | 133 | 111 | 156 | 159 | 120 | 198 | 193 | 56 | 41 |
| | 4 | 136 | 129 | 95 | 177 | 122 | 169 | 204 | 49 | 41 |
| | 5 | 158 | 119 | 63 | 209 | 127 | 29 | 221 | 85 | 167 |
| | 6 | 151 | 135 | 58 | 216 | 138 | 183 | 248 | 59 | 164 |
| | 7 | 129 | 107 | 137 | 158 | 120 | 186 | 187 | 49 | 39 |
| | 8 | 149 | 129 | 75 | 202 | 145 | 10 | 235 | 50 | 164 |
| 1 | 2 | 114 | 98 | 48 | 107 | 78 | 112 | 109 | 77 | 117 |
| 1 | 3 | 109 | 90 | 119 | 108 | 52 | 104 | 93 | 52 | 108 |
| 1 | 4 | 149 | 135 | 84 | 148 | 75 | 103 | 136 | 75 | 102 |
| 1 | 5 | 108 | 95 | 181 | 96 | 39 | 100 | 110 | 31 | 115 |
| 1 | 6 | 180 | 142 | 14 | 147 | 91 | 116 | 185 | 84 | 118 |
| 1 | 7 | 116 | 96 | 105 | 116 | 57 | 105 | 97 | 56 | 113 |
| 1 | 8 | 146 | 117 | 200 | 117 | 65 | 107 | 151 | 54 | 117 |
| 2 | 3 | 140 | 120 | 173 | 127 | 88 | 120 | 137 | 92 | 97 |
| 2 | 4 | 119 | 114 | 166 | 116 | 55 | 108 | 119 | 55 | 109 |
| 2 | 5 | 154 | 148 | 93 | 154 | 80 | 101 | 152 | 71 | 117 |
| 2 | 6 | 128 | 106 | 190 | 108 | 43 | 109 | 131 | 34 | 115 |
| 2 | 7 | 129 | 112 | 157 | 123 | 77 | 120 | 123 | 82 | 99 |
| 2 | 8 | 138 | 126 | 168 | 129 | 50 | 104 | 138 | 44 | 113 |
| 3 | 4 | 88 | 63 | 6 | 64 | 56 | 119 | 94 | 45 | 72 |
| 3 | 5 | 207 | 186 | 164 | 202 | 88 | 102 | 196 | 79 | 114 |
| 3 | 6 | 241 | 211 | 174 | 219 | 119 | 113 | 241 | 116 | 114 |
| 3 | 7 | 23 | 18 | 20 | 21 | 18 | 196 | 26 | 18 | 57 |
| 3 | 8 | 225 | 201 | 156 | 212 | 101 | 107 | 221 | 92 | 115 |
| 4 | 5 | 234 | 225 | 111 | 234 | 98 | 101 | 228 | 91 | 112 |
| 4 | 6 | 239 | 220 | 185 | 222 | 94 | 108 | 243 | 84 | 114 |
| 4 | 7 | 70 | 47 | 196 | 50 | 40 | 134 | 76 | 31 | 72 |
| 4 | 8 | 238 | 227 | 165 | 231 | 89 | 105 | 239 | 79 | 112 |
| 5 | 6 | 137 | 109 | 33 | 118 | 76 | 112 | 136 | 67 | 122 |
| 5 | 7 | 210 | 191 | 120 | 208 | 89 | 102 | 198 | 78 | 116 |
| 5 | 8 | 77 | 58 | 16 | 60 | 46 | 101 | 81 | 36 | 126 |
| 6 | 7 | 232 | 210 | 168 | 219 | 111 | 113 | 232 | 107 | 115 |
| 6 | 8 | 66 | 57 | 42 | 62 | 40 | 116 | 65 | 38 | 122 |
| 7 | 8 | 222 | 203 | 145 | 215 | 97 | 107 | 217 | 86 | 116 |

Table 4.8. The elements of the ellipses computed from $\left(\sigma_{x^i x^j}\right)$ pertaining to the points used in experiment VII.

| | | z = 0 | | | y = 0 | | | x = 0 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| | 1 | 50 | 38 | 98 | 119 | 40 | 17 | 115 | 37 | 2 |
| | 2 | 58 | 52 | 199 | 120 | 40 | 181 | 116 | 58 | 199 |
| | 3 | 87 | 77 | 79 | 116 | 86 | 4 | 137 | 30 | 37 |
| | 4 | 109 | 84 | 33 | 124 | 82 | 173 | 147 | 55 | 44 |
| | 5 | 81 | 75 | 125 | 122 | 76 | 19 | 137 | 33 | 166 |
| | 6 | 104 | 80 | 167 | 122 | 77 | 173 | 142 | 55 | 157 |
| | 7 | 89 | 76 | 70 | 115 | 85 | 191 | 134 | 34 | 37 |
| | 8 | 92 | 73 | 138 | 112 | 86 | 192 | 132 | 37 | 162 |
| 1 | 2 | 68 | 66 | 97 | 68 | 36 | 101 | 66 | 36 | 99 |
| 1 | 3 | 76 | 69 | 135 | 76 | 33 | 112 | 71 | 35 | 96 |
| 1 | 4 | 106 | 96 | 19 | 97 | 41 | 102 | 105 | 41 | 95 |
| 1 | 5 | 72 | 63 | 33 | 68 | 32 | 118 | 71 | 35 | 111 |
| 1 | 6 | 106 | 97 | 180 | 98 | 42 | 101 | 105 | 42 | 103 |
| 1 | 7 | 80 | 75 | 106 | 81 | 34 | 108 | 75 | 35 | 99 |
| 1 | 8 | 86 | 80 | 119 | 86 | 37 | 106 | 80 | 37 | 99 |
| 2 | 3 | 94 | 88 | 154 | 92 | 40 | 111 | 93 | 40 | 89 |
| 2 | 4 | 85 | 73 | 39 | 78 | 30 | 103 | 82 | 30 | 95 |
| 2 | 5 | 100 | 93 | 12 | 94 | 46 | 109 | 102 | 44 | 114 |
| 2 | 6 | 77 | 67 | 161 | 71 | 28 | 103 | 74 | 28 | 103 |
| 2 | 7 | 86 | 81 | 127 | 86 | 35 | 111 | 82 | 36 | 91 |
| 2 | 8 | 84 | 78 | 97 | 85 | 35 | 111 | 79 | 37 | 104 |
| 3 | 4 | 69 | 47 | 193 | 47 | 33 | 110 | 72 | 23 | 78 |
| 3 | 5 | 139 | 135 | 81 | 142 | 57 | 115 | 135 | 64 | 103 |
| 3 | 6 | 161 | 144 | 152 | 154 | 55 | 107 | 154 | 57 | 97 |
| 3 | 7 | 25 | 17 | 196 | 21 | 16 | 172 | 29 | 14 | 61 |
| 3 | 8 | 153 | 137 | 120 | 154 | 55 | 111 | 139 | 60 | 96 |
| 4 | 5 | 167 | 148 | 32 | 153 | 59 | 108 | 163 | 61 | 104 |
| 4 | 6 | 148 | 147 | 36 | 147 | 50 | 103 | 148 | 50 | 99 |
| 4 | 7 | 53 | 32 | 188 | 35 | 25 | 127 | 56 | 18 | 76 |
| 4 | 8 | 159 | 146 | 65 | 157 | 54 | 107 | 150 | 56 | 98 |
| 5 | 6 | 88 | 68 | 196 | 69 | 42 | 96 | 92 | 32 | 120 |
| 5 | 7 | 145 | 136 | 68 | 145 | 57 | 112 | 138 | 61 | 105 |
| 5 | 8 | 52 | 36 | 176 | 40 | 24 | 76 | 53 | 21 | 123 |
| 6 | 7 | 156 | 141 | 140 | 151 | 52 | 107 | 146 | 54 | 98 |
| 6 | 8 | 47 | 34 | 19 | 36 | 27 | 120 | 49 | 21 | 127 |
| 7 | 8 | 153 | 136 | 107 | 154 | 53 | 110 | 136 | 57 | 98 |

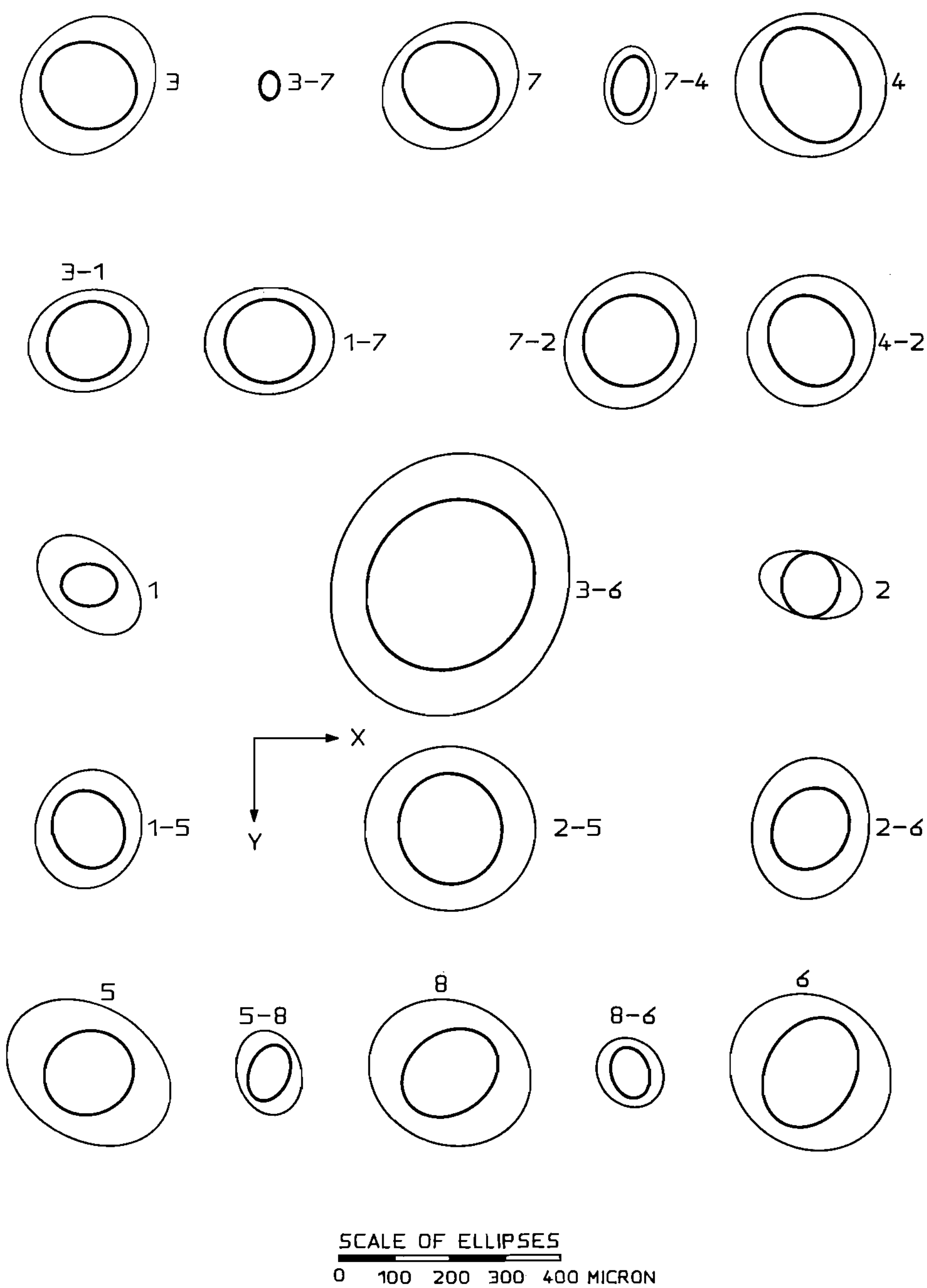


Fig. 4.8. Standard ellipses and relative standard ellipses in the $xy$-plane of experiment VII.

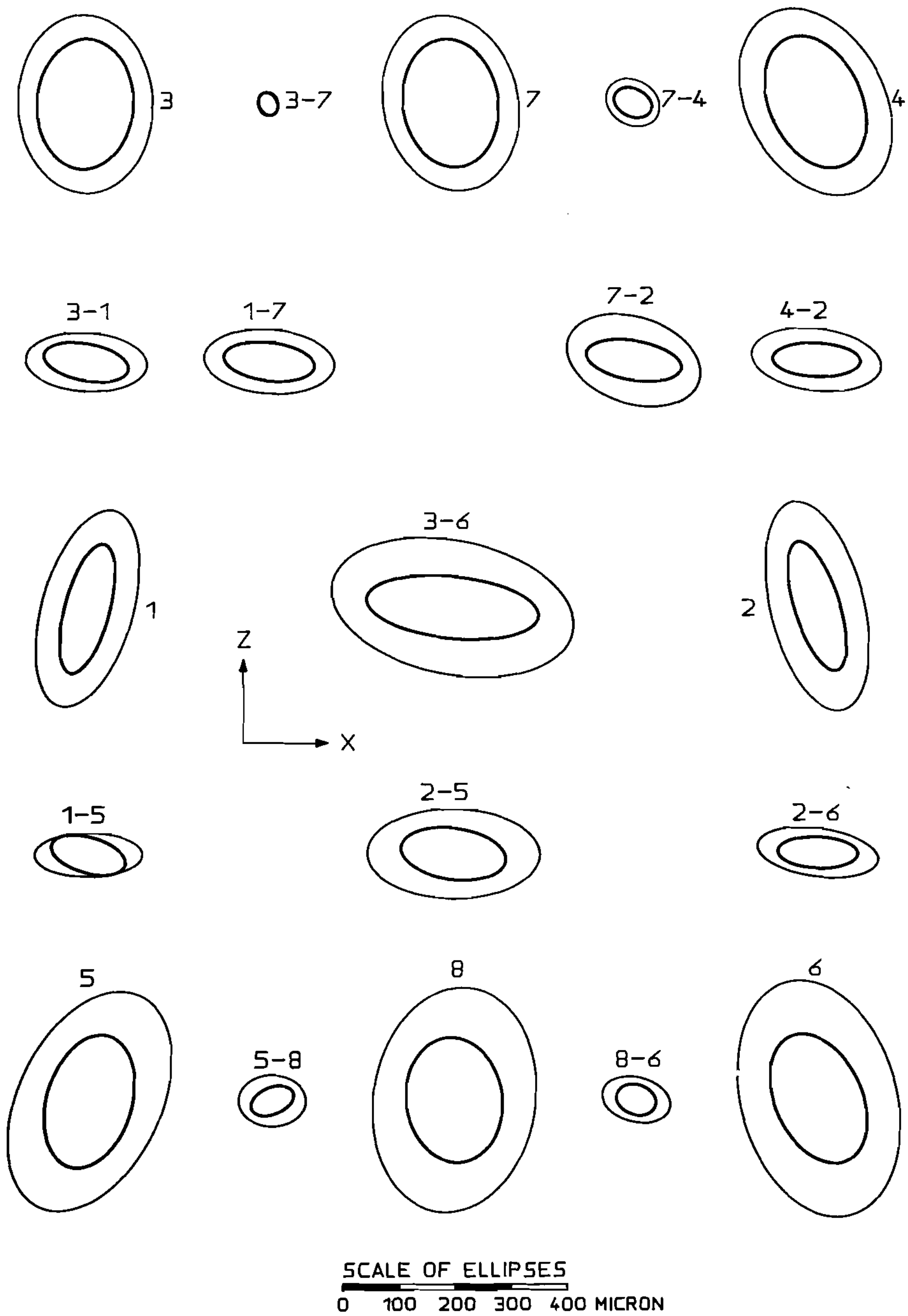


Fig. 4.9. Standard ellipses and relative standard ellipses in the *xz*-plane of experiment VII.

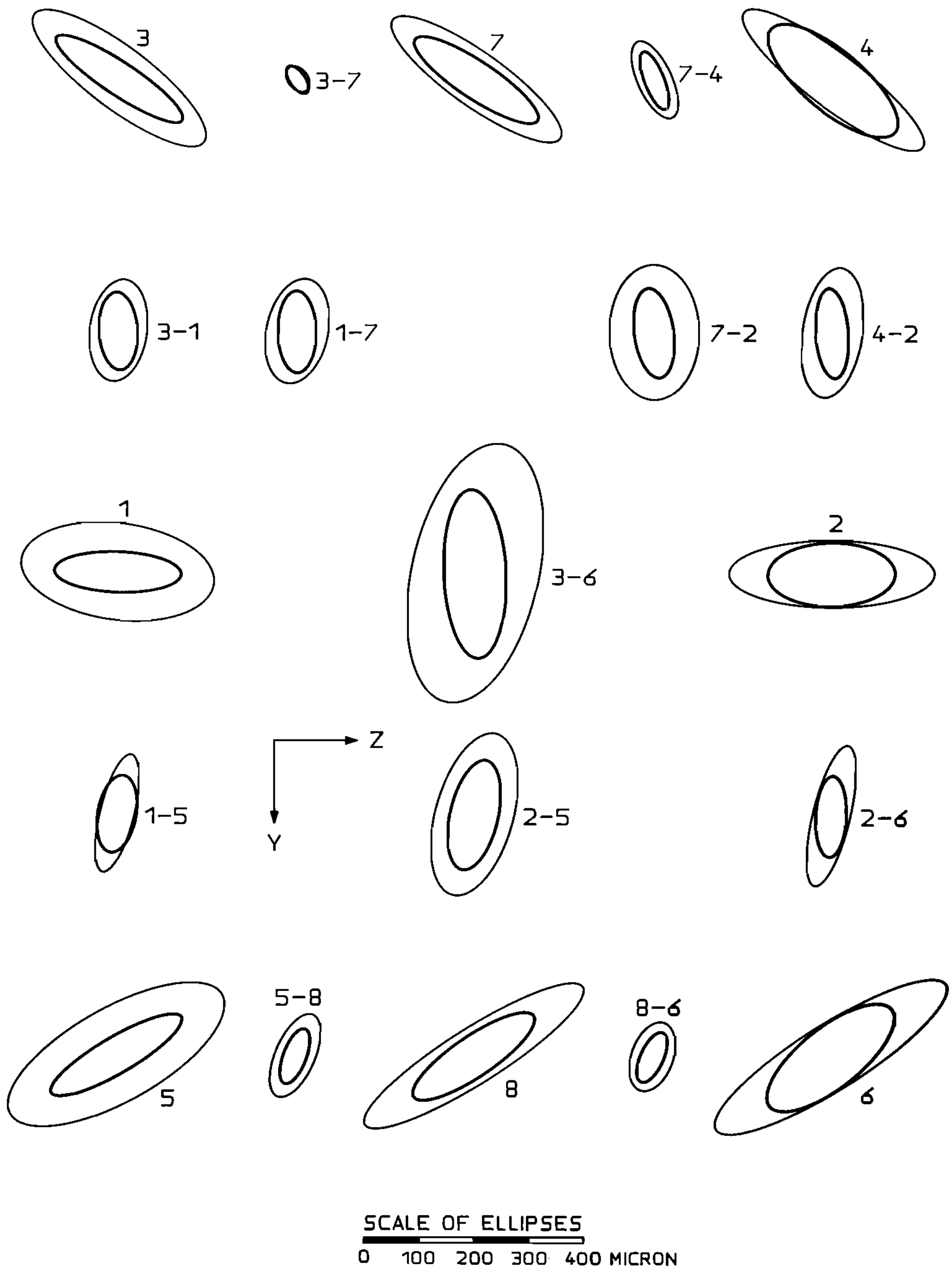


Fig. 4.10. Standard ellipses and relative standard ellipses in the $yz$-plane of experiment VII.

*Experiment VIII*
Stereopair: 116–118
Photo scale: 1:12000
Camera: Wild RC5, $c = 210.38$ mm
Size: $18 \times 18$ cm
Distance model – projection centres: $z \simeq 460$ mm
Instrumental base: $b = 170$ mm

This experiment differs from the previous ones by the following items:
- normal angle photographs
- signalized points
- numerical relative orientation with the Wild A7.

Both inner and relative orientation are repeated 20 times and after each orientation the coordinates of the 8 signalized points are measured in forward and backward sequence. Figures 4.11 and 4.12 give schematically the position of the points.

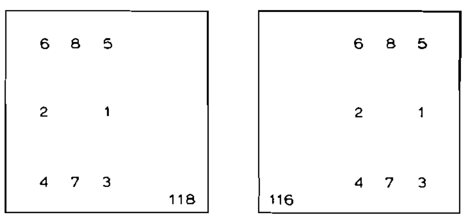


Fig. 4.11. The position of the 8 signalized points on the photographs.

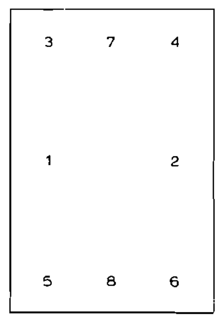


Fig. 4.12. The position of the points in the model.

The estimated covariance matrix and the covariance matrix,

$$\left(\hat{\sigma}_{x^ix^j}\right) \quad \text{and} \quad \left(\sigma_{x^ix^j}\right)$$

computed according to (3.12) and (4.9) are given in table 4.9 respectively in table 4.10.

As in the preceding experiments the shape and position of the ellipsoids are represented by ellipses, see figures 4.13, 4.14 and 4.15; thin lines refer to $\left(\hat{\sigma}_{x^ix^j}\right)$ thick lines refer to $\left(\sigma_{x^ix^j}\right)$.

The elements of the ellipses are given in tables 4.11 and 4.12 respectively.

Table 4.9. The elements of the estimated covariance matrix $\left(\hat{\sigma}_{x^i x^j}\right)$ of experiment VIII.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ | $h_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 843 | | | | | | | | | | | |
| $y_1$ | 463 | 1638 | | | | | | | | | | |
| $h_1$ | −6009 | −7684 | 141259 | | | | | | | | | |
| $x_2$ | 2660 | 2782 | −48976 | 17625 | | | | | | | | |
| $y_2$ | 589 | 1106 | −1624 | 1120 | 1480 | | | | | | | |
| $h_2$ | −5450 | −6901 | 126467 | −44112 | −1693 | 114138 | | | | | | |
| $x_3$ | 930 | 96 | −2857 | 1820 | 858 | −2676 | 1602 | | | | | |
| $y_3$ | −1869 | −1119 | 43376 | −15159 | 379 | 39096 | −1131 | 14933 | | | | |
| $h_3$ | −6136 | −7480 | 133041 | −46810 | −2118 | 119729 | −3079 | 41592 | 128709 | | | |
| $x_4$ | 2331 | 1893 | −34312 | 12860 | 1305 | −31051 | 2183 | −10908 | −33372 | 9985 | | |
| $y_4$ | −1181 | −693 | 34124 | −11618 | 887 | 30628 | −256 | 11788 | 32424 | −7944 | 9768 | |
| $h_4$ | −4819 | −5597 | 105588 | −37171 | −1371 | 95268 | −2372 | 33342 | 101985 | −26481 | 26111 | 81207 |
| $x_5$ | 112 | 46 | 1517 | −354 | −70 | 1334 | −132 | 564 | 1168 | −350 | 259 | 897 |
| $y_5$ | 2595 | 4613 | −60751 | 20979 | 1656 | −54203 | 997 | −17016 | −56557 | 14427 | −13276 | −44558 |
| $h_5$ | −5845 | −7945 | 141305 | −48651 | −1754 | 126135 | −2806 | 42462 | 130884 | −33806 | 33352 | 103932 |
| $x_6$ | 2282 | 3052 | −52956 | 18309 | 455 | −47219 | 804 | −15896 | −49422 | 12552 | −12760 | −39159 |
| $y_6$ | 2791 | 4128 | −57082 | 20105 | 1951 | −51014 | 1831 | −16494 | −53627 | 14457 | −12430 | −42165 |
| $h_6$ | −5221 | −7149 | 132976 | −45591 | −1216 | 118831 | −2305 | 40556 | 123592 | −31609 | 31995 | 98085 |
| $x_7$ | 1775 | 1069 | −19277 | 7672 | 1209 | −17473 | 2069 | −6290 | −19120 | 6434 | −4190 | −15047 |
| $y_7$ | −1600 | −868 | 40807 | −14111 | 717 | 36654 | −779 | 14139 | 38898 | −9971 | 11406 | 31345 |
| $h_7$ | −5728 | −6471 | 122494 | −43162 | −1659 | 110219 | −2972 | 38720 | 118437 | −30844 | 30221 | 94048 |
| $x_8$ | 1640 | 2134 | −34603 | 12109 | 387 | −30782 | 585 | −10270 | −32100 | 8310 | −8276 | −25665 |
| $y_8$ | 2632 | 4168 | −56595 | 19789 | 1768 | −50515 | 1438 | −16147 | −53023 | 13976 | −12386 | −41705 |
| $h_8$ | −5321 | −7453 | 134975 | −46201 | −1283 | 120435 | −2174 | 40731 | 125149 | −31326 | 32300 | 99237 |

Table 4.10. The elements of the covariance matrix $\left(\sigma_{x^i x^j}\right)$ of the points used in experiment VIII.

| | $x_1$ | $y_1$ | $h_1$ | $x_2$ | $y_2$ | $h_2$ | $x_3$ | $y_3$ | $h_3$ | $x_4$ | $y_4$ | $h_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 1934 | | | | | | | | | | | |
| $y_1$ | −36 | 2139 | | | | | | | | | | |
| $h_1$ | 2310 | −3203 | 208082 | | | | | | | | | |
| $x_2$ | 1040 | 951 | −61737 | 20090 | | | | | | | | |
| $y_2$ | 33 | 1912 | 2954 | −881 | 2271 | | | | | | | |
| $h_2$ | 2401 | −2937 | 190786 | −56650 | 2729 | 176286 | | | | | | |
| $x_3$ | 1937 | −83 | 5421 | 137 | 77 | 5256 | 2093 | | | | | |
| $y_3$ | 836 | 861 | 75700 | −22549 | 3104 | 69632 | 1966 | 30326 | | | | |
| $h_3$ | 2357 | −3285 | 213392 | −63550 | 3168 | 196291 | 5542 | 79449 | 223959 | | | |
| $x_4$ | 1139 | 678 | −44024 | 14662 | −580 | −40353 | 501 | −16302 | −45945 | 10986 | | |
| $y_4$ | 727 | 1020 | 60906 | −18196 | 2867 | 56329 | 1638 | 24477 | 63403 | −13057 | 20349 | |
| $h_4$ | 2119 | −2559 | 166244 | −49351 | 2208 | 153272 | 4605 | 61420 | 173146 | −35523 | 49563 | 135354 |
| $x_5$ | 1904 | −219 | 14224 | −2597 | 193 | 13307 | 2137 | 5110 | 14401 | −1451 | 4176 | 11530 |
| $y_5$ | −776 | 3066 | −76156 | 22662 | 851 | −69806 | −1915 | −25271 | −76621 | 15948 | −20138 | −60058 |
| $h_5$ | 1961 | −2964 | 192531 | −57281 | 2525 | 176483 | 4840 | 68715 | 193707 | −40311 | 55576 | 151480 |
| $x_6$ | 1062 | 917 | −59534 | 19354 | −959 | −54560 | 191 | −21451 | −60456 | 13996 | −17371 | −46923 |
| $y_6$ | −887 | 3158 | −77992 | 23313 | 702 | −72043 | −2054 | −26388 | −79952 | 16499 | −21124 | −62355 |
| $h_6$ | 2391 | −2920 | 189678 | −56256 | 3027 | 174651 | 5231 | 68371 | 192738 | −39547 | 55493 | 150458 |
| $x_7$ | 1588 | 304 | −19724 | 7568 | −278 | −17907 | 1314 | −7361 | −20745 | 5857 | −5842 | −15822 |
| $y_7$ | 874 | 876 | 71691 | −21301 | 2949 | 66022 | 1946 | 28600 | 74958 | −15380 | 23258 | 58227 |
| $h_7$ | 2381 | −3007 | 195317 | −58019 | 2762 | 179874 | 5300 | 72444 | 204216 | −41893 | 58081 | 158640 |
| $x_8$ | 1391 | 529 | −34360 | 11880 | −519 | −31331 | 898 | −12320 | −34722 | 8738 | −9949 | −26891 |
| $y_8$ | −937 | 3116 | −75226 | 22378 | 767 | −69407 | −2063 | −25185 | −76561 | 15742 | −20153 | −59817 |
| $h_8$ | 2414 | −2958 | 192129 | −56909 | 2878 | 176675 | 5291 | 68994 | 194493 | −39977 | 55945 | 152103 |

| $h_4$ | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 81207 | | | | | | | | | | | | |
| 897 | 468 | | | | | | | | | | | |
| −44558 | − 578 | 27908 | | | | | | | | | | |
| 103932 | 1826 | −61841 | 143474 | | | | | | | | | |
| −39159 | − 236 | 23125 | −53232 | 20351 | | | | | | | | |
| −42165 | − 834 | 25650 | −57717 | 21294 | 24656 | | | | | | | |
| 98085 | 1804 | −57435 | 134117 | −49929 | −54105 | 126645 | | | | | | |
| −15047 | − 272 | 8008 | −18940 | 6898 | 8547 | −17521 | 4609 | | | | | |
| 31345 | 467 | −15855 | 39992 | −15039 | −15237 | 38290 | − 5525 | 13641 | | | | |
| 94048 | 1114 | −51667 | 120648 | −45470 | −49132 | 113877 | −17739 | 36349 | 109391 | | | |
| −25665 | 62 | 15227 | −34675 | 13417 | 13995 | −32487 | 4588 | − 9747 | − 29771 | 9056 | | |
| −41705 | − 674 | 25632 | −57315 | 21332 | 24195 | −53579 | 8042 | −14999 | − 48557 | 14042 | 23980 | |
| 99237 | 1655 | −58772 | 136572 | −50972 | −54843 | 128361 | −17559 | 38449 | 115307 | −33211 | −54581 | 130722 |

| $h_4$ | $x_5$ | $y_5$ | $h_5$ | $x_6$ | $y_6$ | $h_6$ | $x_7$ | $y_7$ | $h_7$ | $x_8$ | $y_8$ | $h_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 135354 | | | | | | | | | | | | |
| 11530 | 2678 | | | | | | | | | | | |
| − 60058 | − 5233 | 30535 | | | | | | | | | | |
| 151480 | 13227 | −72438 | 183134 | | | | | | | | | |
| − 46923 | − 2484 | 22153 | − 55993 | 19027 | | | | | | | | |
| − 62355 | − 5359 | 30459 | − 72311 | 22642 | 31945 | | | | | | | |
| 150458 | 13335 | −70268 | 177650 | −54974 | −72274 | 176230 | | | | | | |
| − 15822 | 369 | 7131 | − 18025 | 7252 | 7372 | − 17498 | 3747 | | | | | |
| 58227 | 4923 | −23922 | 65196 | −20245 | −24938 | 64781 | − 6906 | 27277 | | | | |
| 158640 | 13408 | −70258 | 177625 | −55143 | −73198 | 176496 | −18810 | 68731 | 187254 | | | |
| − 26891 | − 711 | 12896 | − 32596 | 11672 | 13012 | − 31610 | 4926 | −11603 | − 31603 | 7559 | | |
| − 59817 | − 5272 | 29662 | − 70297 | 21846 | 30698 | − 69792 | 6971 | −23815 | − 70137 | 12522 | 29886 | |
| 152103 | 13540 | −71518 | 180810 | −55637 | −72742 | 177889 | −17662 | 65418 | 178229 | −31996 | −70701 | 180793 |

Table 4.11. The elements of the ellipses computed from $\left(\hat{\sigma}_{x^i x^j}\right)$ pertaining to experiment VIII.

| | | $z = 0$ | | | $y = 0$ | | | $x = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ | $a$ | $b$ | $\psi$ |
| | 1 | 43 | 25 | 27 | 376 | 24 | 197 | 376 | 35 | 197 |
| | 2 | 133 | 37 | 96 | 362 | 22 | 176 | 338 | 38 | 199 |
| | 3 | 123 | 39 | 195 | 359 | 39 | 198 | 377 | 37 | 20 |
| | 4 | 133 | 44 | 150 | 300 | 35 | 180 | 300 | 35 | 20 |
| | 5 | 167 | 21 | 199 | 379 | 21 | 1 | 413 | 32 | 174 |
| | 6 | 210 | 33 | 47 | 383 | 24 | 176 | 387 | 36 | 174 |
| | 7 | 128 | 45 | 172 | 335 | 41 | 190 | 349 | 37 | 21 |
| | 8 | 180 | 25 | 34 | 373 | 24 | 184 | 392 | 32 | 175 |
| 1 | 2 | 116 | 25 | 109 | 121 | 31 | 78 | 53 | 23 | 174 |
| 1 | 3 | 137 | 23 | 4 | 62 | 24 | 198 | 138 | 60 | 108 |
| 1 | 4 | 135 | 25 | 162 | 126 | 40 | 38 | 149 | 44 | 148 |
| 1 | 5 | 144 | 26 | 191 | 46 | 33 | 9 | 143 | 46 | 103 |
| 1 | 6 | 182 | 37 | 49 | 130 | 40 | 91 | 136 | 40 | 91 |
| 1 | 7 | 135 | 25 | 183 | 78 | 39 | 19 | 139 | 58 | 125 |
| 1 | 8 | 152 | 29 | 34 | 82 | 44 | 91 | 132 | 43 | 93 |
| 2 | 3 | 173 | 36 | 50 | 127 | 55 | 89 | 128 | 52 | 86 |
| 2 | 4 | 104 | 23 | 24 | 78 | 24 | 168 | 112 | 42 | 136 |
| 2 | 5 | 208 | 42 | 156 | 143 | 61 | 80 | 169 | 54 | 120 |
| 2 | 6 | 150 | 35 | 6 | 61 | 27 | 170 | 151 | 50 | 111 |
| 2 | 7 | 139 | 34 | 38 | 85 | 52 | 118 | 117 | 55 | 102 |
| 2 | 8 | 152 | 35 | 185 | 64 | 49 | 187 | 152 | 54 | 115 |
| 3 | 4 | 89 | 22 | 81 | 113 | 19 | 53 | 81 | 21 | 22 |
| 3 | 5 | 278 | 47 | 197 | 102 | 48 | 3 | 278 | 99 | 106 |
| 3 | 6 | 299 | 58 | 29 | 143 | 90 | 107 | 269 | 90 | 99 |
| 3 | 7 | 47 | 14 | 86 | 55 | 18 | 60 | 36 | 14 | 19 |
| 3 | 8 | 279 | 51 | 20 | 105 | 87 | 147 | 267 | 96 | 101 |
| 4 | 5 | 270 | 49 | 177 | 151 | 73 | 39 | 273 | 80 | 126 |
| 4 | 6 | 248 | 53 | 13 | 122 | 44 | 167 | 255 | 76 | 120 |
| 4 | 7 | 45 | 16 | 72 | 62 | 20 | 43 | 53 | 16 | 24 |
| 4 | 8 | 242 | 49 | 0 | 118 | 45 | 188 | 256 | 80 | 122 |
| 5 | 6 | 146 | 34 | 105 | 148 | 36 | 89 | 49 | 27 | 162 |
| 5 | 7 | 276 | 51 | 187 | 110 | 71 | 18 | 276 | 93 | 113 |
| 5 | 8 | 98 | 22 | 108 | 99 | 27 | 88 | 35 | 21 | 168 |
| 6 | 7 | 276 | 60 | 21 | 121 | 69 | 140 | 264 | 87 | 107 |
| 6 | 8 | 51 | 16 | 100 | 51 | 24 | 90 | 28 | 11 | 170 |
| 7 | 8 | 263 | 54 | 10 | 107 | 51 | 169 | 262 | 92 | 109 |

Table 4.12. The elements of the ellipses computed from $\left(\sigma_{x^i x^j}\right)$ pertaining to the points used in experiment VIII.

| | | z = 0 | | | y = 0 | | | x = 0 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | a | b | ψ | a | b | ψ | a | b | ψ |
| | 1 | 46 | 44 | 189 | 456 | 44 | 1 | 456 | 46 | 199 |
| | 2 | 142 | 47 | 103 | 441 | 41 | 180 | 420 | 47 | 1 |
| | 3 | 175 | 44 | 4 | 473 | 44 | 2 | 502 | 44 | 22 |
| | 4 | 172 | 42 | 161 | 381 | 39 | 183 | 392 | 44 | 23 |
| | 5 | 177 | 42 | 189 | 429 | 41 | 5 | 460 | 40 | 176 |
| | 6 | 221 | 44 | 41 | 440 | 41 | 181 | 454 | 44 | 175 |
| | 7 | 171 | 43 | 183 | 435 | 43 | 194 | 461 | 42 | 23 |
| | 8 | 188 | 44 | 27 | 432 | 43 | 189 | 457 | 44 | 176 |
| 1 | 2 | 142 | 20 | 106 | 146 | 38 | 83 | 54 | 22 | 187 |
| 1 | 3 | 175 | 10 | 2 | 73 | 12 | 1 | 177 | 69 | 91 |
| 1 | 4 | 176 | 16 | 160 | 140 | 46 | 49 | 169 | 53 | 138 |
| 1 | 5 | 165 | 11 | 190 | 79 | 27 | 192 | 165 | 75 | 90 |
| 1 | 6 | 215 | 17 | 44 | 142 | 60 | 82 | 170 | 61 | 86 |
| 1 | 7 | 173 | 11 | 182 | 71 | 47 | 21 | 168 | 65 | 109 |
| 1 | 8 | 180 | 15 | 30 | 90 | 56 | 64 | 163 | 62 | 88 |
| 2 | 3 | 219 | 22 | 47 | 158 | 68 | 75 | 174 | 61 | 75 |
| 2 | 4 | 135 | 18 | 18 | 80 | 21 | 169 | 140 | 49 | 126 |
| 2 | 5 | 242 | 23 | 152 | 167 | 80 | 98 | 177 | 79 | 106 |
| 2 | 6 | 181 | 20 | 199 | 59 | 14 | 183 | 181 | 57 | 101 |
| 2 | 7 | 179 | 22 | 34 | 94 | 61 | 94 | 155 | 59 | 92 |
| 2 | 8 | 185 | 21 | 179 | 65 | 58 | 144 | 175 | 60 | 103 |
| 3 | 4 | 115 | 25 | 81 | 155 | 34 | 49 | 120 | 18 | 20 |
| 3 | 5 | 334 | 11 | 196 | 140 | 22 | 198 | 337 | 132 | 90 |
| 3 | 6 | 368 | 23 | 25 | 156 | 106 | 65 | 344 | 106 | 89 |
| 3 | 7 | 57 | 18 | 90 | 72 | 28 | 53 | 55 | 13 | 18 |
| 3 | 8 | 344 | 17 | 16 | 129 | 83 | 21 | 337 | 114 | 89 |
| 4 | 5 | 328 | 18 | 175 | 149 | 96 | 54 | 309 | 103 | 114 |
| 4 | 6 | 309 | 29 | 7 | 111 | 21 | 176 | 312 | 88 | 112 |
| 4 | 7 | 60 | 22 | 71 | 88 | 24 | 40 | 78 | 18 | 24 |
| 4 | 8 | 302 | 25 | 196 | 109 | 32 | 196 | 307 | 91 | 113 |
| 5 | 6 | 163 | 39 | 98 | 163 | 63 | 97 | 73 | 18 | 167 |
| 5 | 7 | 333 | 12 | 186 | 123 | 75 | 194 | 325 | 123 | 99 |
| 5 | 8 | 108 | 33 | 102 | 108 | 48 | 98 | 56 | 16 | 164 |
| 6 | 7 | 342 | 27 | 16 | 105 | 88 | 175 | 331 | 101 | 97 |
| 6 | 8 | 57 | 20 | 93 | 57 | 35 | 91 | 38 | 16 | 174 |
| 7 | 8 | 325 | 21 | 6 | 109 | 35 | 191 | 324 | 107 | 98 |

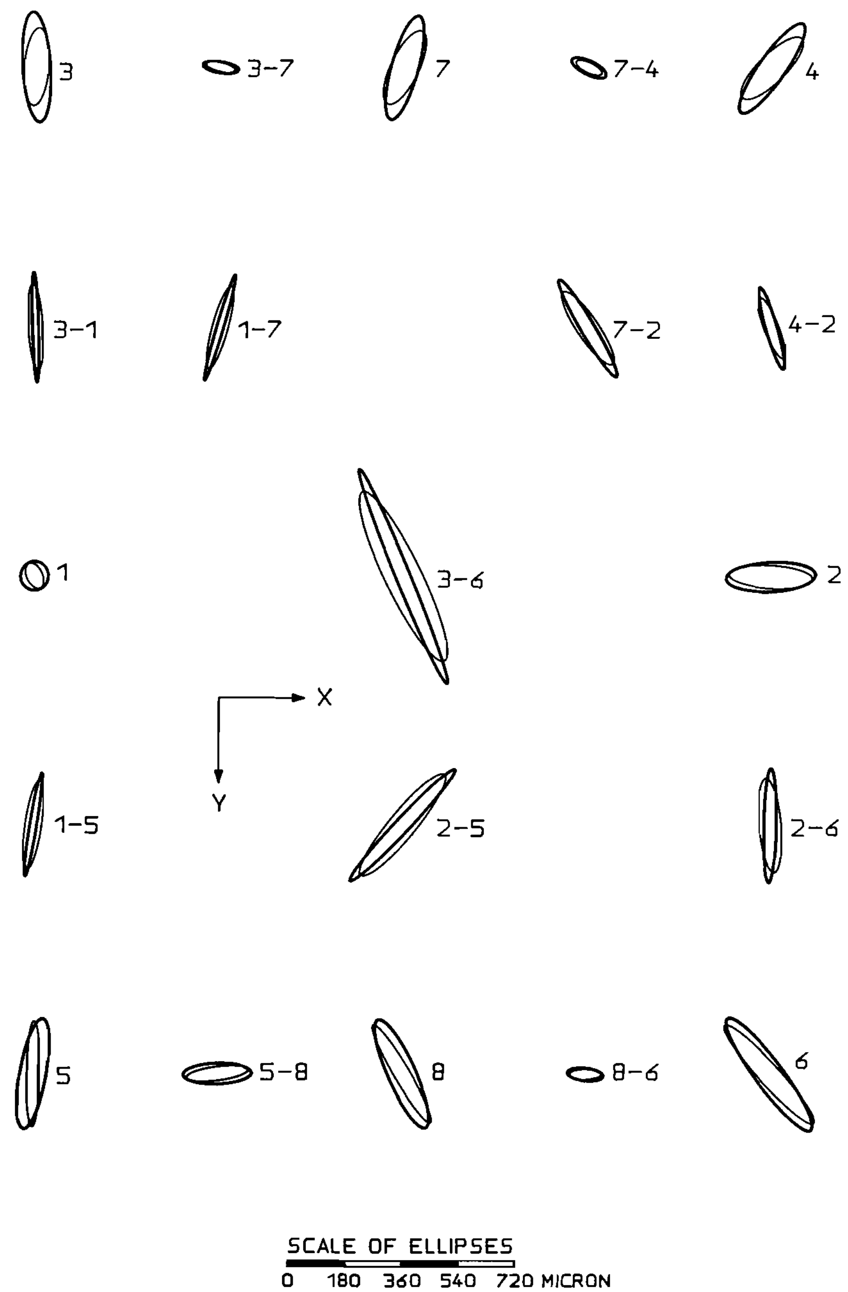


Fig. 4.13. Standard ellipses and relative standard ellipses in the $xy$-plane of experiment VIII.

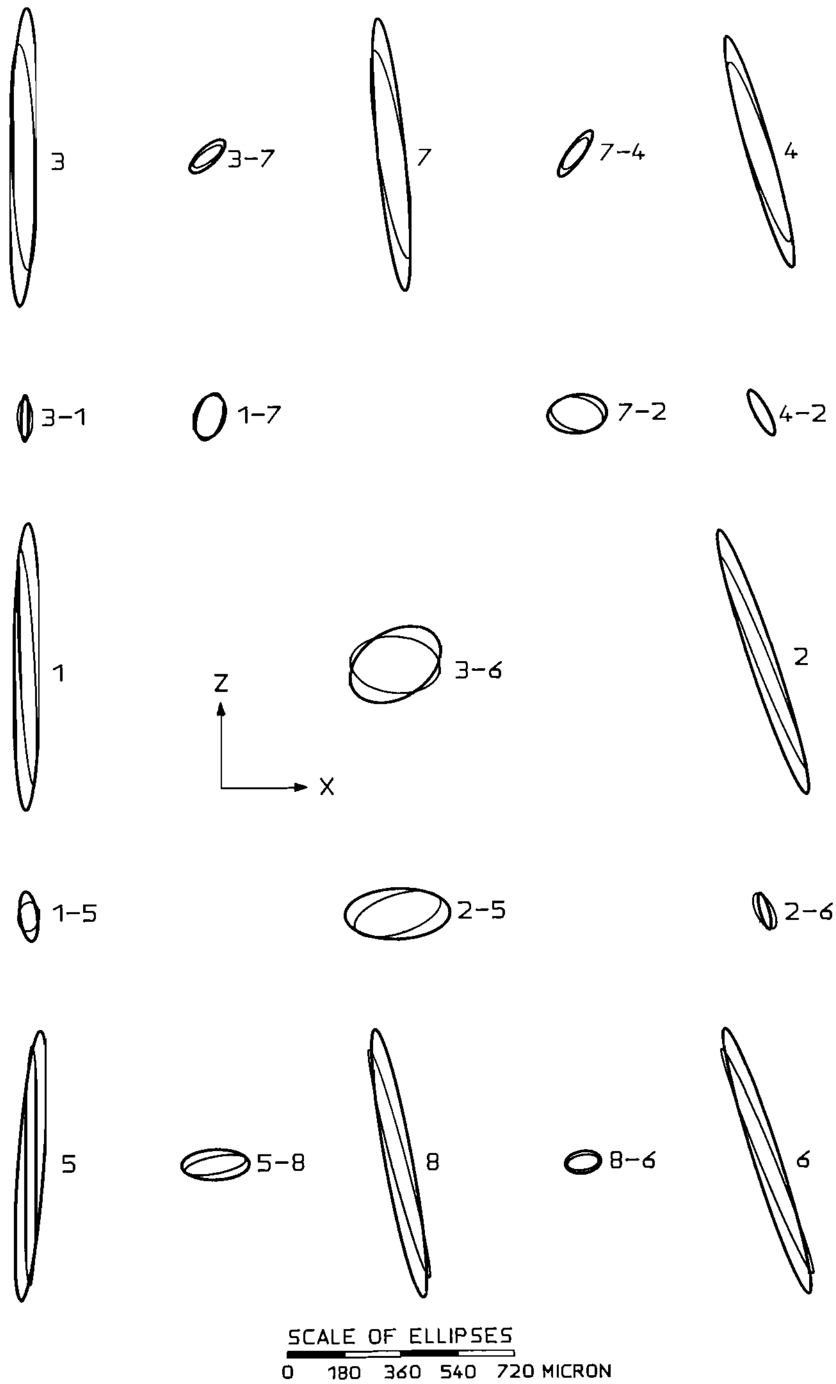


Fig. 4.14. Standard ellipses and relative standard ellipses in the $xz$-plane of experiment VIII.

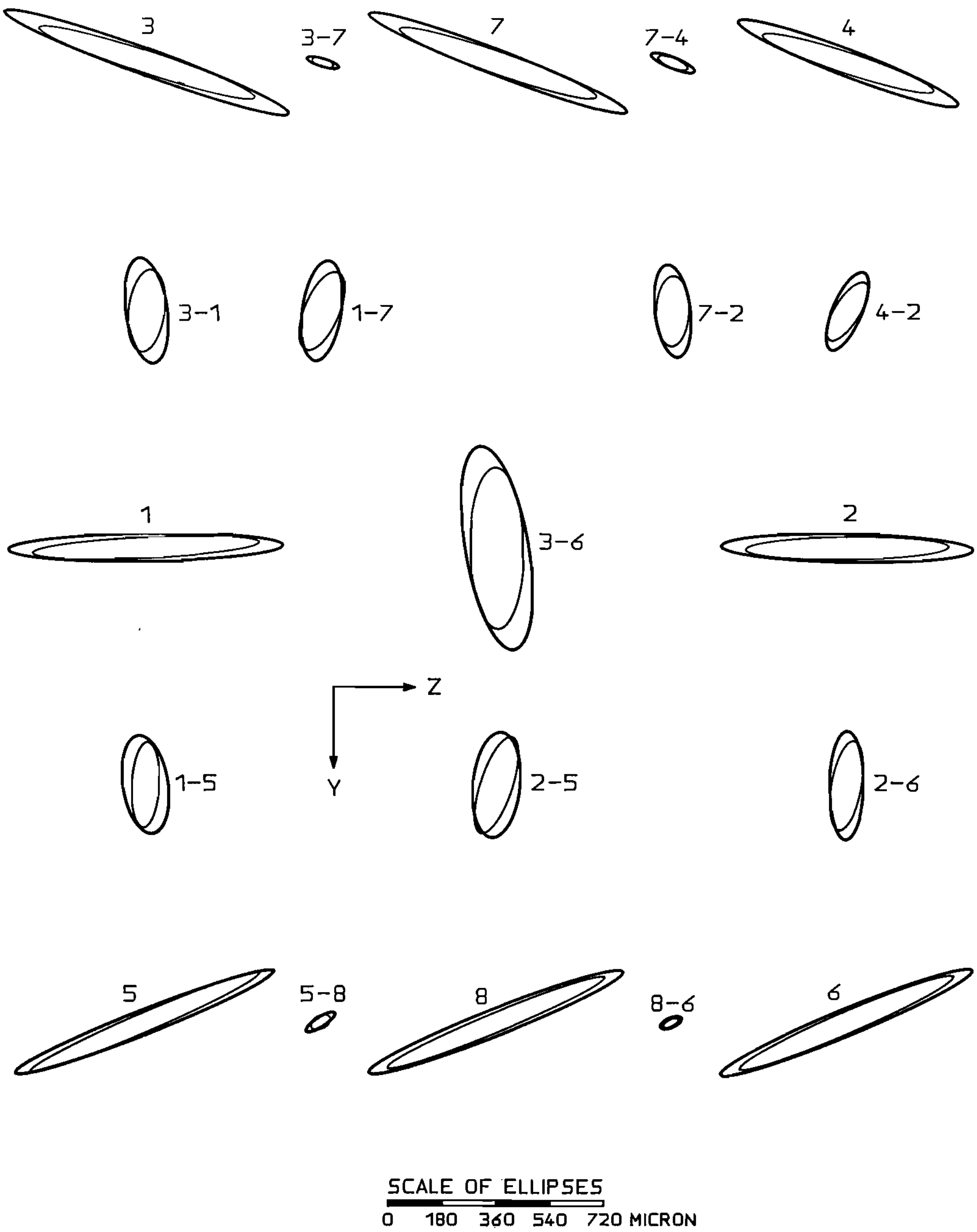


Fig. 4.15. Standard ellipses and relative standard ellipses in the *yz*-plane of experiment VIII.

# V RECAPITULATION AND CONCLUSIONS

The influence of the random observation errors on the coordinates of model points has been represented by the covariance matrix of the coordinates determined in two different ways:

1. by executing repeated measurements of points in photogrammetric models; in some cases these repeated measurements are combined with repeated relative orientations or with repeated relative and inner orientations; from the series of these measurements the estimated covariance matrices ($\hat{\sigma}^2$) have been determined.
2. by writing the machine coordinates as functions of the initial observations, e.g. $x$- and $y$-parallaxes; by means of the standard deviations of the initial observations and applying the law of propagation of errors the covariance matrices ($\sigma^2$) have been computed.

The eight experiments, I to VIII, executed to compare ($\hat{\sigma}^2$) and ($\sigma^2$) can be divided into three groups, group *a*, *b* and *c*. Per group a single type or a combination of different types of observation errors is studied:

group *a*: the measuring of the coordinates of a model point; experiment I and II.
group *b*: the measuring of the coordinates of a model point together with the relative orientation; experiment III, IV and V.
group *c*: the measuring of the coordinates of a model point, the relative orientation and the inner orientation; experiment VI, VII and VIII.

In each group the experiments can be distinguished by:

- pricked points or signalized points
- wide angle (W.A.) or normal angle (N.A.)-photographs
- numerical or empirical relative orientation
- the autograph Wild A7 or Wild A8

In table 5.1 the experiments are conveniently arranged.
The estimated covariance matrix ($\hat{\sigma}^2$) is computed from the 20 repeated measurements of the coordinates of model points per experiment. This makes 8 full-matrices of $24 \times 24$ elements for the $8 \times 3$ coordinates of the 8 model points.

The eight covariance matrices ($\sigma^2$) are computed from the standard observation of the individual observations such as $x$- and $y$-parallaxes by applying the law of propagation of errors. A recapitulation of the standard deviations of the individual observations can be given as follows:

The standard deviations of the observations for measuring a model point, given in photo scale, are according (2.5):

| | $\sigma_{\Delta p_x}$ | $\sigma_{x'}$ | $\sigma_{y'}$ |
|---|---|---|---|
| signalized points | 4.9 | 4.7 | 6.0 |
| pricked points | 6.5 | 4.2 | 6.5 |

Table 5.1. A survey of the experiments.

| group | experiment | observations errors studied: measuring of a point | relative orientation | inner orientation | pricked points | signalized points | W.A. photo's | N.A. photo's | num. rel. orientation | emp. rel. orientation | Wild A7 | Wild A8 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *a* | I | × | | | × | | × | | | | | × |
| | II | × | | | × | | × | | | | | × |
| *b* | III | × | × | | × | | × | | × | | × | |
| | IV | × | × | | × | | × | | | × | | × |
| | V | × | × | | | × | | × | | × | | × |
| *c* | VI | × | × | × | × | | × | | × | | × | |
| | VII | × | × | × | × | | × | | | × | | × |
| | VIII | × | × | × | | × | | × | × | | × | |

The standard deviation of the observation for relative orientation, the $y$-parallax, given in photo scale, is, see (3.9):

Wild A7: $\sigma_{p_y} = 9$ micron
Wild A8: $\sigma_{p_y} = 11$ micron

The standard deviations of the observations for the elements of inner orientation of both camera's are, see (4.7):

$$\sigma_{x'} = \sigma_{x''} = \sigma_{y'} = \sigma_{y''} = 20 \text{ micron}$$

$$\sigma_{c'} = \sigma_{c''} = 3 \text{ micron}$$

These standard deviations are introduced for the computation of covariance matrix $(\sigma^2)$ of the 8 experiments. In all experiments the same values are used. More practical experience will however be necessary in order to gain a better insight into the variance of the observations, the factors influencing these values, etc.

The standard ellipsoids and relative standard ellipsoids, represented by sub-matrices of $(\hat{\sigma}^2)$ and $(\sigma^2)$, are given in a large number of diagrams showing the projections of the ellipsoids. The projections are standard ellipses and relative standard ellipses. Some general remarks and conclusions will be made now on the basis of the drawn ellipses.

The observation errors in group $a$, the measuring of a model point, are relatively small and for that the ellipses of experiment I and II are drawn on a larger scale than those of the following experiments. The observations are assumed to be correlation free and therefore the relative standard ellipses are left out of consideration.

The coordinates $z$ and $x$ and the coordinates $x$ and $y$ of a model point are undoubtedly correlated. This correlation determined from the repeated measurements, the $(\hat{\sigma}^2)$, agrees with the correlation computed from the initial observations, the $(\sigma^2)$. Compare the thin line ellipses of $(\hat{\sigma}^2)$ with the thick line ellipses of $(\sigma^2)$ in figures 2.4, 2.5 and 2.6 and in figures 2.9, 2.10 and 2.11.

The experiments III, IV and V in group *b* refer to the observation errors: measuring a model point, and relative orientation. For these three experiments the following remarks hold good; see figures 3.3, 3.4 and 3.5 of experiment III, figures 3.8, 3.9 and 3.10 of experiment IV and figures 3.13, 3.14 and 3.15 of experiment V.

- the coordinates of a single point are strongly correlated.
- the coordinates of different points are strongly correlated, especially for those at shorter distances.
- the ellipses of experiment V are larger than those of III and IV because normal angle photographs are used in this experiment; in experiments III and IV wide angle photographs have been used.
- it can be proved that the scale of the ellipses is mainly determined by the standard deviation of the *y*-parallax, $\sigma_{p_y}$; here only two groups are distinguished: Wild A7 and Wild A8, or probably better said: numerical relative orientation and empirical relative orientation; more detailed study will be necessary to study how far this standard deviation is influenced by observer, instrument, photographs, etc.

The experiments VI, VII and VIII of group *c* refer to the observation errors: measuring a model point, relative orientation and inner orientation. The results are given in respectively figures 4.3, 4.4 and 4.5, figures 4.8, 4.9 and 4.10 and figures 4.13, 4.14 and 4.15.

For these three experiments broadly the same remarks can be made as for the experiments of group *b*. The ellipses in group *c* are in general slightly larger in consequence of the influence of the inner orientation.

It can be noticed for both groups *b* and *c* that the ellipses of the covariance matrix $(\hat{\sigma}^2)$ of an experiment are sometimes smaller and sometimes larger than those of $(\sigma^2)$. This could not be explained by this small number of experiments; possibly different observation errors must be applied for these experiments or it is mainly due to the small number of observations, only 20 repeated measurements per experiment. Furthermore it is matter of course that these scale differences of the ellipses are greater for the Wild A8 measurements, due to the method of relative orientation, the empirical method; see experiment VII. But the important property of correlation between the coordinates of one point and the coordinates of different points corresponds very well between the two matrices $(\hat{\sigma}^2)$ and $(\sigma^2)$.

This investigation shows that the covariance matrix $(\sigma^2)$ gives a good description of the influence of random observation errors on the coordinates of model points. This holds only when the measurements and observations are carried out as has been described here. More detailed studies will be needed to verify how far generalization or differentiation is necessary if more models are jointed to a strip or a block.

The structure of the covariance matrix $(\sigma^2)$ is essential for studies of precision and accuracy where photogrammetric models are used as a basic unit.

# APPENDIX 1

The machine coordinates of a model point, $x$, $y$ and $h$, are influenced by observation errors. In this appendix a formula will be derived which gives the relation between the machine coordinates of a point and the quantities which are observed when setting the floating mark onto a model point.

The observations are the setting of the mark in the proper elevation and in the proper planimetric position. These observations can further be defined as:

the horizontal parallax: $\Delta p_x$
the $x$-setting : $\Delta x'$
the $y$-setting : $\Delta y'$

The error in elevation setting influences the planimetric position. This relation is demonstrated in figure 1. Point $P$ is a model point, an intersection point of corresponding rays, whose coordinates are to be measured. Point $O$, the middle of the base $O_L O_R = b$, is the origin of the $xyz$-system.

We assume that, in the presence of an error in elevation, the floating mark is set in (see figure 1):

$M_L$: if point $P$ is marked on the left photo
$M_R$: if point $P$ is marked on the right photo
$M$ : if $P$ is a natural point or a signalized point

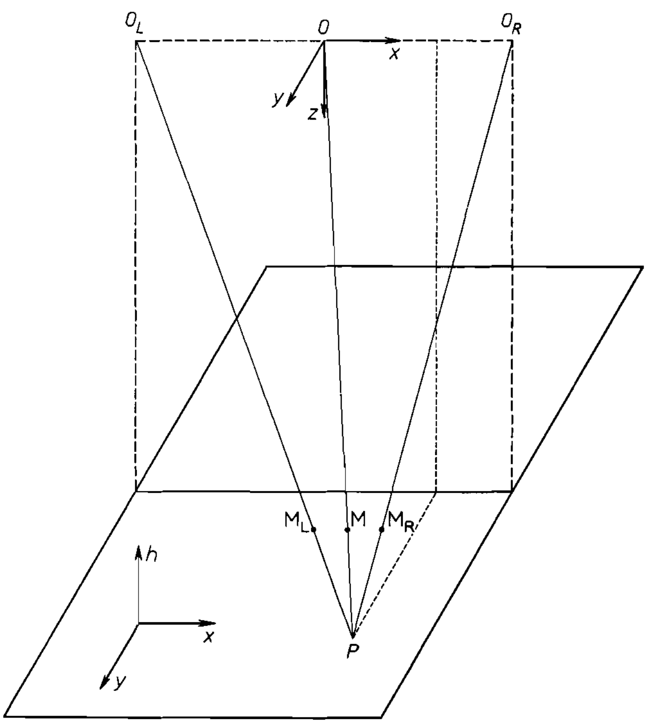


Fig. 1. The influence of the error in elevation on the planimetric position.

The following principle underlies this assumption. If point $P$ is a marked point, the floating mark is set in such a way that its projection on the photograph coincides with that marked point. That is to say if $P$ is marked on the left respectively on the right photo, the mark will be set on the line $PO_L$ respectively $PO_R$. In case $P$ is a natural or signalized point the mark is assumed to be set in $x$- and $y$-direction in such a way that its projections on the photographs are symmetrical with respect to the images of point $P$. In this case the mark is actually set on the line $PO$, which connects point $P$ and the middle of the base $O$.

The following coordinate differences are introduced:

$$\Delta\bar{x}_{ML} = x^{\mathrm{ML}} - x \quad \ldots \quad (1)$$

$$\Delta\bar{x}_{MR} = x^{\mathrm{MR}} - x \quad \ldots \quad (2)$$

$$\Delta\bar{x}_{M} = x^{\mathrm{M}} - x \quad \ldots \quad (3)$$

$$\Delta\bar{y}_{M} = y^{\mathrm{ML}} - y = y^{\mathrm{MR}} - y = y^{\mathrm{M}} - y \quad \ldots \quad (4)$$

$$\Delta h_{M} = z - z^{\mathrm{ML}} = z - z^{\mathrm{MR}} = z - z^{\mathrm{M}} \quad \ldots \quad (5)$$

where:

$x\ ,y\ ,z$ are the coordinates of $P$,
$x^{\mathrm{ML}}, y^{\mathrm{ML}}, z^{\mathrm{ML}}$ are the coordinates of $M_L$,
$x^{\mathrm{MR}}, y^{\mathrm{MR}}, z^{\mathrm{MR}}$ are the coordinates of $M_R$ and
$x^{\mathrm{M}}\ , y^{\mathrm{M}}\ , z^{\mathrm{M}}$ are the coordinates of $M$.

$$\Delta p_x = x^{\mathrm{MR}} - x^{\mathrm{ML}} \quad \ldots \quad (6)$$

We read from figure 1:

$$\Delta\bar{x}_{ML} = -\frac{x+\frac{1}{2}b}{z}\Delta h_M \quad \ldots \quad (7)$$

$$\Delta\bar{x}_{MR} = -\frac{x-\frac{1}{2}b}{z}\Delta h_M \quad \ldots \quad (8)$$

$$\Delta\bar{x}_{M} = -\frac{x}{z}\Delta h_M \quad \ldots \quad (9)$$

$$\Delta\bar{y}_{M} = -\frac{y}{z}\Delta h_M \quad \ldots \quad (10)$$

$$\Delta h_{M} = \frac{z}{b}\Delta p_x \quad \ldots \quad (11)$$

These formulae give only the relation between the differentials of the machine coordinates and the differential of height, or the differential of the horizontal parallax.

Adding the differentials $\Delta x'$ and $\Delta y'$ of the proper $x$- and $y$-setting of the floating mark we have:

$$\Delta x_{ML} = \Delta \bar{x}_{ML} + \Delta x' \quad \ldots \quad (12)$$

$$\Delta x_{MR} = \Delta \bar{x}_{MR} + \Delta x' \quad \ldots \quad (13)$$

$$\Delta x_{M} = \Delta \bar{x}_{M} + \Delta x' \quad \ldots \quad (14)$$

$$\Delta y_{M} = \Delta \bar{y}_{M} + \Delta y' \quad \ldots \quad (15)$$

Introducing (7), (8), (9) and (10) in respectively (12), (13), (14) and (15) and applying (11) makes in matrix-notation:

$$\begin{bmatrix} \Delta x_{ML} \\ \Delta x_{MR} \\ \Delta x_{M} \\ \Delta y_{M} \\ \Delta h_{M} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2b}(2x+b) & 1 & 0 \\ -\frac{1}{2b}(2x-b) & 1 & 0 \\ -\frac{x}{b} & 1 & 0 \\ -\frac{y}{b} & 0 & 1 \\ \frac{z}{b} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta p_x \\ \Delta x' \\ \Delta y' \end{bmatrix} \quad \ldots \quad (16)$$

This formula gives the relation between the differentials of the machine coordinates of the model point $P$ and the differentials of the observed quantities:
the horizontal parallax: $\Delta p_x$
the $x$-setting : $\Delta x'$
the $y$-setting : $\Delta y'$

For the differentials in the $x$-coordinate we have distinghuished 3 cases:
index $ML$: point $P$ pricked on left photo
index $MR$: point $P$ pricked on right photo
index $M$ : natural or signalized point

The differentials of the $y$- and $h$-coordinate are the same for the 3 cases.

# APPENDIX 2

In order to perform a relative orientation with two perspective bundles of rays $y$-parallaxes have to be measured or eliminated. The influence of the observation errors in $y$-parallaxes on the orientation elements is a known problem.

Starting from the well-known parallax foımula (the signs applying to a Wild A7):

$$\Delta p_y = +\frac{y^2+z^2}{z}\Delta\omega_2 - \frac{(2x-b)y}{2z}\Delta\varphi_2 + \frac{2x-b}{2}\Delta\varkappa_2 - \frac{y}{z}\Delta bz_2 - \Delta by_2 \tag{1}$$

the matrix of weight coefficients of the orientation elements can be found by the formulae of least square adjustment; standard problem II. Assuming numerical relative orientation and parallax measurements in the 6 points 1 to 6, see figure 1, the result is:

$$\overline{(\Delta O),(\Delta O)^T} = \overline{\begin{pmatrix} \Delta\omega_2 \\ \Delta\varphi_2 \\ \Delta\varkappa_2 \\ \Delta bz_2 \\ \Delta by_2 \end{pmatrix}, \begin{pmatrix} \Delta\omega_2 \\ \Delta\varphi_2 \\ \Delta\varkappa_2 \\ \Delta bz_2 \\ \Delta by_2 \end{pmatrix}^T} =$$

$$= \begin{pmatrix} +\frac{3z^2}{4a^4} & 0 & 0 & 0 & +\frac{z(2a^2+3z^2)}{4a^4} \\ 0 & +\frac{z^2}{a^2b^2} & 0 & +\frac{z^2}{2a^2b} & 0 \\ 0 & 0 & +\frac{2}{3b} & 0 & -\frac{1}{3b} \\ 0 & +\frac{z^2}{2a^2b} & 0 & +\frac{z^2}{2a^2} & 0 \\ +\frac{z(2a^2+3z^2)}{4a^4} & 0 & -\frac{1}{3b} & 0 & +\frac{8a^4+12a^2z^2+9z^4}{12a^4} \end{pmatrix} \tag{2}$$

In figure 1 the quantities $a$ and $b$ are indicated.
The covariance matrix of the orientation elements is:

$$(\sigma_{OO}) = \sigma_{p_y}^2 \overline{(\Delta O),(\Delta O)^T} \quad \dots\dots\dots\dots \tag{3}$$

$\sigma_{p_y}$ is the standard deviation of the $y$-parallax observation.

In order to find the matrix of weight coefficients of machine coordinates in relation to the $y$-parallax observations, first the differential formula of machine coordinates and orientation elements must be known.

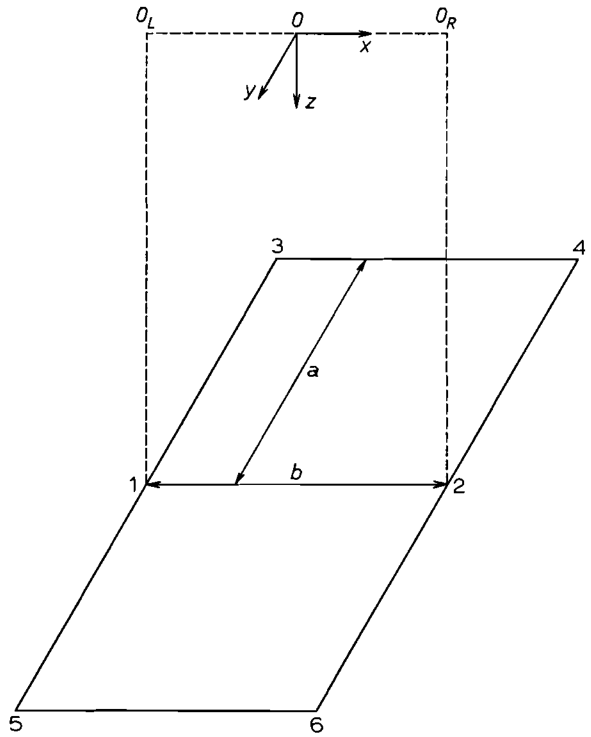


Fig. 1. A photogrammetric model with the quantities $a$ and $b$.

Figures 2 and 3 show the differential change in position of the model point, i.e. the intersection of two corresponding rays, $P$ to $P'$ in consequence of a small variation in the orientation of the two bundles.

Figure 2 gives the projection in the $xz$-plane and in figure 3 the projection in the $yz$-plane.

The small variations of the orientation of the bundles bring about small displacements of the intersection of the ray with the horizontal plane through $P$ or $P'$; see figures 2 and 3.

for the left bundle:

$$\Delta x_1 \quad \text{and} \quad \Delta y_1 \qquad \qquad (4a)$$

for the right bundle:

$$\Delta x_2 \quad \text{and} \quad \Delta y_2 \qquad \qquad (4b)$$

Besides that figure 3 shows a height difference $\Delta z_0$ corresponding with a difference in $y$-coordinate:

$$\Delta \bar{y} \qquad \qquad (4c)$$

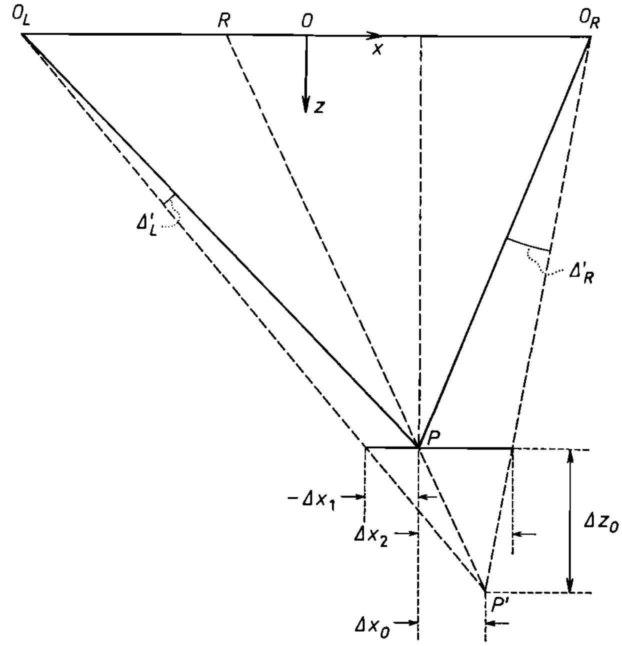


Fig. 2. The differential change $P$ to $P'$ in the $xz$-plane.

In case corresponding rays intersect only approximately we define the model point according to the usual principal of symmetry in the theory of stereoscopic vision: the model point is the middle of the line connecting the two corresponding rays in $y$-direction, $P'$ in figures 2 and 3; in figure 3 is $P'_L P' = P'P'_R$; in case of assymmetrical points, for example points pricked on one photograph, the model point is the end of the horizontal connection line in $y$-direction, $P'_L$ for points pricked on the left photograph and $P'_R$ for points pricked on the right photograph; see figure 3.

The small variations of the coordinates of model point $P$ are:

$$\Delta x_O, \Delta y_O, \Delta y_{OL}, \Delta y_{OR} \quad \text{and} \quad \Delta z_O \qquad (5)$$

Three cases are distinguished for the $y$-coordinate:
$\Delta y_O$ : natural or signalized points
$\Delta y_{OL}$: point $P$ is pricked on left photo
$\Delta y_{OR}$: point $P$ is pricked on right photo

The differentials of the $x$-and $z$-coordinate are the same for the three cases. The index $O$ is introduced here as these variations are caused by variations in the orientation elements.

The differentials of the machine coordinates of model point $P$ in (5) can easily be expressed in:

$$\Delta x_1, \Delta x_2, \Delta y_1, \Delta y_2 \quad \text{and} \quad \Delta \bar{y}$$

as defined in (4).

The coordinates of $P$ are: $x$, $y$ and $z$; the origin of the axes is chosen in the middle of the base, $O$; see figure 3.

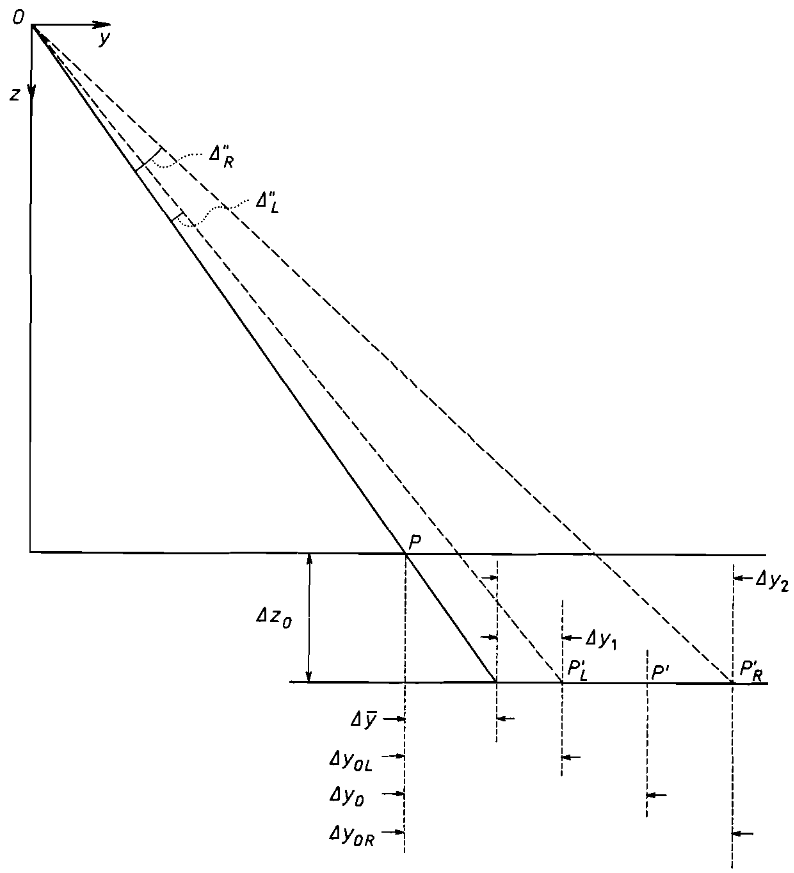


Fig. 3. The differential change of $P$ to $P'_L$, $P'$ and $P'_R$ in the $yz$-plane.

The auxiliary line $RPP'$ and $P\bar{P}$ in figure 2 shows:

$$\frac{\Delta z_O}{\Delta x_O} = \frac{z}{\frac{1}{2}b + x - O_L R} \qquad (6)$$

$$\frac{-\Delta x_1}{\Delta x_2} = \frac{O_L R}{b - O_L R} \qquad (7)$$

and

$$\frac{\Delta z_O}{\Delta x_2 - \Delta x_1} = \frac{z}{b} \qquad (8)$$

From figure 3 we read:

$$\frac{\Delta \bar{y}}{\Delta z_O} = \frac{y}{z} \qquad (9)$$

$$\Delta y_O = \Delta \bar{y} + \tfrac{1}{2}(\Delta y_1 + \Delta y_2) \qquad (10)$$

$$\Delta y_{OL} = \Delta \bar{y} + \Delta y_1 \quad (11)$$

$$\Delta y_{OR} = \Delta \bar{y} + \Delta y_2 \quad (12)$$

It follows from (6), (7) and (8) that:

$$\Delta x_O = \frac{2x+b}{2b}(\Delta x_2 - \Delta x_1) + \Delta x_1 \quad (13)$$

$$\Delta z_O = \frac{z}{b}(\Delta x_2 - \Delta x_1) \quad (14)$$

and from (8), (9), (10), (11) and (12) it follows that:

$$\Delta y_O = \frac{y}{b}(\Delta x_2 - \Delta x_1) + \tfrac{1}{2}(\Delta y_1 + \Delta y_2) \quad (15)$$

$$\Delta y_{OL} = \frac{y}{b}(\Delta x_2 - \Delta x_1) + \Delta y_1 \quad (16)$$

$$\Delta y_{OR} = \frac{y}{b}(\Delta x_2 - \Delta x_1) + \Delta y_2 \quad (17)$$

The differential formulae of $\Delta x_1$, $\Delta x_2$, $\Delta y_1$ and $\Delta y_2$ as defined in (4) and the orientation elements of the left and the right camera and suitable for A7 and A8 measurements, are the well-known formulae:

$$\Delta x_1 = +\frac{(2x+b)y}{2z}\Delta\omega_1 - \frac{(2x+b)^2+4z^2}{4z}\Delta\varphi_1 - y\,\Delta\varkappa_1 - \frac{2x+b}{2z}\Delta bz_1 \quad (18)$$

$$\Delta x_2 = +\frac{(2x-b)y}{2z}\Delta\omega_2 - \frac{(2x-b)^2+4z^2}{4z}\Delta\varphi_2 - y\,\Delta\varkappa_2 - \frac{2x-b}{2z}\Delta bz_2 \quad (19)$$

$$\Delta y_1 = +\frac{y^2+z^2}{z}\Delta\omega_1 - \frac{(2x+b)y}{2z}\Delta\varphi_1 + \frac{2x+b}{2}\Delta\varkappa_1 - \frac{y}{z}\Delta bz_1 - \Delta by_1 \quad (20)$$

$$\Delta y_2 = +\frac{y^2+z^2}{z}\Delta\omega_2 - \frac{(2x-b)y}{2z}\Delta\varphi_2 + \frac{2x-b}{2}\Delta\varkappa_2 - \frac{y}{z}\Delta bz_2 - \Delta by_2 \quad (21)$$

Further it is evident that:

$$\Delta z_O = -\Delta h_O \quad (22)$$

For the relative orientation with an A7 the following orientation elements are supposed to be used:

$$\Delta\omega_2, \Delta\varphi_2, \Delta\varkappa_2, \Delta bz_2 \quad \text{and} \quad \Delta by_2 \qquad (23)$$

then:

$$\Delta\omega_1 = \Delta\varphi_1 = \Delta\varkappa_1 = \Delta bz_1 = \Delta by_1 = 0 \qquad (24)$$

We obtain by substitution of (18), (19) and (24) in (13):

$$\Delta x_O = +\frac{(2x+b)(2x-b)y}{4bz}\Delta\omega_2 - \frac{2x+b}{b}\cdot\frac{(2x-b)+4z^2}{4z}\Delta\varphi_2 + \\ -\frac{(2x+b)y}{2b}\Delta\varkappa_2 - \frac{(2x+b)(2x-b)}{4bz}\Delta bz_2 \qquad (25)$$

Substitution of (18), (19), (20), (21) and (24) in (15) gives:

$$\Delta y_O = +\frac{2xy^2+bz^2}{2bz}\Delta\omega_2 - \frac{(2x-b)xy+2yz^2}{2bz}\Delta\varphi_2 + \left(\frac{2x-b}{4} - \frac{y^2}{b}\right)\Delta\varkappa_2 + \\ -\frac{xy}{bz}\Delta bz_2 - \tfrac{1}{2}\Delta by_2 \qquad (26)$$

Similarly (18), (19), (20) and (24) in (16):

$$\Delta y_{OL} = +\frac{(2x-b)y^2}{2bz}\Delta\omega_2 - \frac{(2x-b)^2y+4z^2y}{4bz}\Delta\varphi_2 - \frac{y^2}{b}\Delta\varkappa_2 - \frac{(2x-b)y}{2bz}\Delta bz_2 \qquad (27)$$

and (18), (19), (21) and (24) in (17):

$$\Delta y_{OR} = +\frac{(2x+b)y+2bz^2}{2bz}\Delta\omega_2 - \frac{(2x+b)(2x-b)y+4yz^2}{4bz}\Delta\varphi_2 + \\ +\left(\frac{2x-b}{2} - \frac{y^2}{b}\right)\Delta\varkappa_2 - \frac{(2x+b)y}{2bz}\Delta bz_2 - \Delta by_2 \qquad (28)$$

and finally (18), (19), (22) and (24) in (14):

$$\Delta h_O = -\frac{(2x-b)y}{2b}\Delta\omega_2 + \frac{(2x-b)^2+4z^2}{4b}\Delta\varphi_2 + \frac{yz}{b}\Delta\varkappa_2 + \frac{2x-b}{2b}\Delta bz_2 \qquad (29)$$

In general matrix notation (25) to (29) is:

$$\begin{pmatrix} \Delta x_O \\ \Delta y_O \\ \Delta y_{OL} \\ \Delta y_{OR} \\ \Delta h_O \end{pmatrix} = (A_O^i) \begin{pmatrix} \Delta\omega_2 \\ \Delta\varphi_2 \\ \Delta\varkappa_2 \\ \Delta bz_2 \\ \Delta by_2 \end{pmatrix} \qquad (30a)$$

with:

$$(A_O^i) \equiv \begin{pmatrix} +\frac{(2x+b)(2x-b)y}{4bz} & -\frac{2x+b}{b}\cdot\frac{(2x-b)^2+4z^2}{4z} & -\frac{(2x+b)y}{2b} & -\frac{(2x+b)(2x-b)}{4bz} & 0 \\ +\frac{2xy^2+bz^2}{2bz} & -\frac{(2x-b)xy+2yz^2}{2bz} & +\left(\frac{2x-b}{4}-\frac{y^2}{b}\right) & -\frac{xy}{bz} & -\frac{1}{2} \\ +\frac{(2x-b)y^2}{2bz} & -\frac{(2x-b)^2y+4z^2y}{4bz} & -\frac{y^2}{b} & -\frac{(2x-b)y}{2bz} & 0 \\ +\frac{(2x+b)y^2+2bz^2}{2bz} & -\frac{(2x+b)(2x-b)y+4yz^2}{4bz} & +\left(\frac{2x-b}{2}-\frac{y^2}{b}\right) & -\frac{(2x+b)y}{2bz} & -1 \\ -\frac{(2x-b)y}{2b} & +\frac{(2x-b)^2+4z^2}{4b} & +\frac{yz}{b} & +\frac{2x-b}{2b} & 0 \end{pmatrix} \tag{30b}$$

In the same way formulae suited to A8 measurements can be derived. The parallax formula is:

$$\Delta p_y = +\frac{y^2+z^2}{z}\Delta\omega_2 - \frac{(2x-b)y}{2z}\Delta\varphi_2 + \frac{(2x+b)y}{2z}\Delta\varphi_1 + \frac{2x-b}{2}\Delta\varkappa_2 - \frac{2x+b}{2}\Delta k_1 \tag{31}$$

and from this formula the weight coefficients of the orientation elements can be derived:

$$\overline{(\Delta O),(\Delta O)^T} = \overline{\begin{pmatrix} \Delta\omega_2 \\ \Delta\varphi_2 \\ \Delta\varphi_1 \\ \Delta\varkappa_2 \\ \Delta\varkappa_1 \end{pmatrix},\begin{pmatrix} \Delta\omega_2 \\ \Delta\varphi_2 \\ \Delta\varphi_1 \\ \Delta\varkappa_2 \\ \Delta\varkappa_1 \end{pmatrix}^T} =$$

$$= \begin{pmatrix} +\frac{3z^2}{4a^4} & 0 & 0 & +\frac{z(3z^2+2a^2)}{4a^4b} & +\frac{z(3z^2+2a^2)}{4a^4b} \\ 0 & +\frac{z^2}{2a^2b^2} & 0 & 0 & 0 \\ 0 & 0 & +\frac{z^2}{2a^2b^2} & 0 & 0 \\ +\frac{z(3z^2+2a^2)}{4a^4b} & 0 & 0 & +\frac{8a^4+12a^2z^2+9z^4}{12a^4b^2} & +\frac{4a^4+12a^2z^2+9z^4}{12a^4b^2} \\ +\frac{z(3z^2+2a^2)}{4a^4b} & 0 & 0 & +\frac{4a^4+12a^2z^2+9z^4}{12a^4b^2} & +\frac{8a^4+12a^2z^2+9z^4}{12a^4b^2} \end{pmatrix} \tag{32}$$

The relative orientation with a Wild A8 is mostly done empirically. We assume here for the empirical method of relative orientation the same propagation of errors as for the numerical method. Although this is an approximation, experiments showed that this approximation satisfied very well.

The covariance matrix of the orientation elements is:

$$(\sigma_{OO}) = \sigma_{p_y}^2 \overline{(\Delta O), (\Delta O)^T} \quad \ldots \ldots \ldots \ldots \quad (33)$$

$\sigma_{p_y}$ is the standard deviation of the $y$-parallax observation.

The differential formulae which give the relation between the machine coordinates and the orientation elements can be derived in the same way as those for a Wild A7 in the previous part.

The orientation elements to be used are:

$$\Delta\omega_2, \Delta\varphi_2, \Delta\varphi_1, \Delta\varkappa_2 \quad \text{and} \quad \Delta\varkappa_1 \quad \ldots \ldots \ldots \ldots \quad (34)$$

and for that we get instead of (24):

$$\Delta\omega_1 = \Delta by_2 = \Delta by_1 = \Delta bz_2 = \Delta bz_1 = 0 \quad \ldots \ldots \ldots \ldots \quad (35)$$

From (13) to (22) and (35) we obtain by substitution:

$$\begin{pmatrix} \Delta x_O \\ \Delta y_O \\ \Delta y_{OL} \\ \Delta y_{OR} \\ \Delta h_O \end{pmatrix} = (A_O^i) \begin{pmatrix} \Delta\omega_2 \\ \Delta\varphi_2 \\ \Delta\varphi_1 \\ \Delta\varkappa_2 \\ \Delta\varkappa_1 \end{pmatrix} \qquad (36a)$$

with:

$$\begin{pmatrix}
+\frac{(2x+b)(2x-b)y}{4bz} & -\frac{(2x+b)}{2b}\cdot\frac{(2x-b)^2+4z^2}{4z} & +\frac{(2x-b)}{2b}\cdot\frac{(2x+b)^2+4z^2}{4z} & -\frac{(2x+b)y}{2b} & +\frac{(2x-b)y}{2b} \\
+\frac{2xy^2+bz^2}{2bz} & -\frac{(2x-b)xy+2yz^2}{2bz} & +\frac{(2x+b)xy+2yz^2}{2bz} & +\left(\frac{2x-b}{4}-\frac{y^2}{b}\right) & +\left(\frac{2x+b}{4}+\frac{y^2}{b}\right) \\
+\frac{(2x-b)y^2}{2bz} & -\frac{(2x-b)^2y+4yz^2}{4bz} & +\frac{(2x+b)(2x-b)y+4yz^2}{4bz} & -\frac{y^2}{b} & +\left(\frac{2x+b}{2}+\frac{y^2}{b}\right) \\
+\frac{(2x+b)y^2+2bz^2}{2bz} & -\frac{(2x+b)(2x-b)y+4yz^2}{4bz} & +\frac{(2x+b)^2y+4yz^2}{4bz} & +\left(\frac{2x-b}{2}-\frac{y^2}{b}\right) & +\frac{y^2}{b} \\
-\frac{(2x-b)y}{2b} & +\frac{(2x-b)^2+4z^2}{4b} & -\frac{(2x+b)^2+4z^2}{4b} & +\frac{yz}{b} & -\frac{yz}{b}
\end{pmatrix} \equiv (A_O^i) \qquad (36b)$$

## APPENDIX 3

The problem of the inner orientation has three variables for each camera, the translations of the projection centre in three mutually perpendicular directions, the elements of the inner orientation:

$$\text{left camera: } \Delta x', \Delta y' \quad \text{and} \quad \Delta c' \qquad (1)$$

$$\text{right camera: } \Delta x'', \Delta y'' \quad \text{and} \quad \Delta c'' \qquad (2)$$

Small variations of the elements of inner orientation correspond with small displacements of the intersection of a ray with a horizontal plane:

for the left bundle, see figure 1:

$$\Delta x_1 \quad \text{and} \quad \Delta y_1 \qquad (3)$$

likewise for the right bundle:

$$\Delta x_2 \quad \text{and} \quad \Delta y_2 \qquad (4)$$

From figure 1 follows the relation between (1) and (3):

$$\Delta x_1 = -\frac{z}{c}\Delta x' - \frac{2x+b}{2c}\Delta c' \qquad (5)$$

$$\Delta y_1 = -\frac{z}{c}\Delta y' - \frac{y}{c}\Delta c' \qquad (6)$$

In the same way we find for the right camera:

$$\Delta x_2 = -\frac{z}{c}\Delta x'' - \frac{2x-b}{2c}\Delta c'' \qquad (7)$$

$$\Delta y_2 = -\frac{z}{c}\Delta y'' - \frac{y}{c}\Delta c'' \qquad (8)$$

The relations between the small displacements of the intersection of a ray, as defined in (3) and (4), and the machine coordinates are already mentioned in appendix 2 formulae (13) to (17):

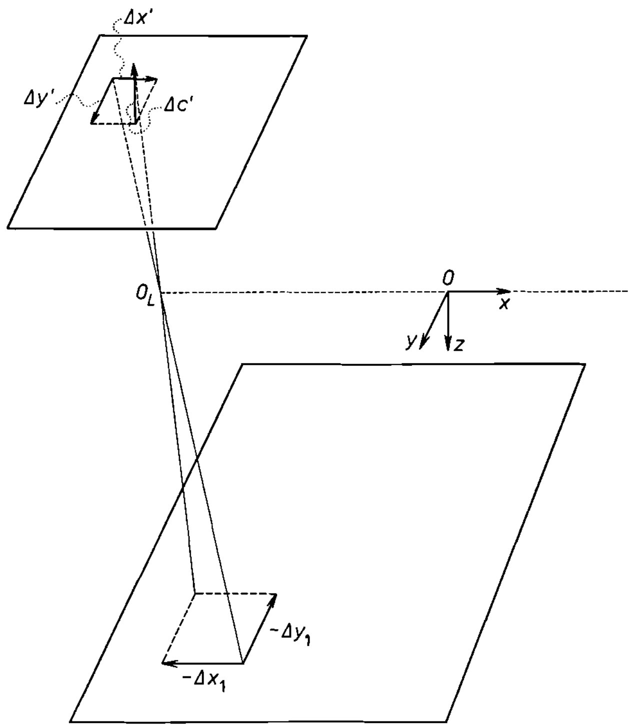


Fig. 1. The differential changes of the elements of inner orientation.

$$\Delta x_I = \frac{2x+b}{2b}(\Delta x_2 - \Delta x_1) + \Delta x_1 \quad \ldots \quad (9)$$

$$\Delta y_I = \frac{y}{b}(\Delta x_2 - \Delta x_1) + \tfrac{1}{2}(\Delta y_1 + \Delta y_2) \quad \ldots \quad (10)$$

$$\Delta y_{IL} = \frac{y}{b}(\Delta x_2 - \Delta x_1) + \Delta y_1 \quad \ldots \quad (11)$$

$$\Delta y_{IR} = \frac{y}{b}(\Delta x_2 - \Delta x_1) + \Delta y_2 \quad \ldots \quad (12)$$

$$\Delta z_I = \frac{z}{b}(\Delta x_2 - \Delta x_1) \quad \ldots \quad (13)$$

The index $I$, used here, indicates that these variations of the machine coordinates are caused by variations of the inner orientation. Substitution of (5) to (8) in (9) to (13) gives:

$$\Delta x_I = \frac{z(2x-b)}{2bc}\Delta x' - \frac{z(2x+b)}{2bc}\Delta x'' + \frac{(2x+b)(2x-b)}{4bc}\Delta c' - \frac{(2x+b)(2x-b)}{4bc}\Delta c'' \quad \ldots \quad (14)$$

$$\Delta y_I = +\frac{yz}{bc}\Delta x' - \frac{yz}{bc}\Delta x'' - \frac{z}{2c}\Delta y' - \frac{z}{2c}\Delta y'' + \frac{xy}{bc}\Delta c' - \frac{xy}{bc}\Delta c'' \quad \ldots\ldots (15)$$

$$\Delta y_{IL} = +\frac{yz}{bc}\Delta x' - \frac{yz}{bc}\Delta x'' - \frac{z}{c}\Delta y' + \frac{y(2x-b)}{2bc}\Delta c' - \frac{y(2x-b)}{2bc}\Delta c'' \quad \ldots (16)$$

$$\Delta y_{IR} = +\frac{yz}{bc}\Delta x' - \frac{yz}{bc}\Delta x'' - \frac{z}{c}\Delta y'' + \frac{y(2x+b)}{2bc}\Delta c' - \frac{y(2x+b)}{2bc}\Delta c'' \quad \ldots (17)$$

$$\Delta z_I = +\frac{z^2}{bc}\Delta x' - \frac{z^2}{bc}\Delta x'' + \frac{z(2x+b)}{2bc}\Delta c' - \frac{z(2x-b)}{2bc}\Delta c'' \quad \ldots\ldots (18)$$

The variations of the inner orientation elements generate a *y*-parallax, $\Delta p_y$. In accordance with (3) and (4) we can write:

$$\Delta p_y = \Delta y_1 - \Delta y_2 \quad \ldots\ldots (19)$$

whence by introducing (6) and (8):

$$\Delta p_y = -\frac{z}{c}\Delta y' + \frac{z}{c}\Delta y'' - \frac{y}{c}\Delta c' + \frac{y}{c}\Delta c'' \quad \ldots\ldots (20)$$

Comparing (20) with formula (1) of appendix 2 which gives the relation between *y*-parallax and orientation elements of an A7:

$$\Delta p_y = -\frac{y}{z}\Delta bz_2 - \Delta by_2 \quad \ldots\ldots (21)$$

it is evident that the *y*-parallax caused by variations of the inner orientation can be eliminated in one horizontal plane, for example mean level $z = z_0$, by changing $by_2$ and $bz_2$.

From (20) and (21) follows:

$$\Delta by_2 = \frac{z_0}{c}\Delta y' - \frac{z_0}{c}\Delta y'' \quad \ldots\ldots (22)$$

$$\Delta bz_2 = \frac{z_0}{c}\Delta c' - \frac{z_0}{c}\Delta c'' \quad \ldots\ldots (23)$$

As a consequence of these corrections of the orientation elements the coordinates of model points change, see (25) to (29) of appendix 2:

$$\Delta x_I = -\frac{(2x+b)(2x-b)}{4bz}\Delta bz_2 \quad \ldots\ldots (24)$$

$$\Delta y_I = -\frac{xy}{bz}\Delta bz_2 - \tfrac{1}{2}\Delta by_2 \quad \ldots\ldots (25)$$

$$\Delta y_{IL} = -\frac{(2x-b)y}{2bz}\Delta bz_2 \quad \ldots\ldots (26)$$

$$\Delta y_{IR} = -\frac{(2x+b)y}{2bz}\Delta bz_2 - \Delta by_2 \quad \ldots\ldots (27)$$

$$\Delta z_I = -\frac{2x-b}{2b}\Delta bz_2 \quad \ldots\ldots (28)$$

introducing (22) and (23):

$$\Delta x_I = -\frac{(2x+b)(2x-b)z_0}{4bcz}\Delta c' + \frac{(2x+b)(2x-b)z_0}{4bcz}\Delta c'' \quad \ldots\ldots (29)$$

$$\Delta y_I = -\frac{z_0}{2c}\Delta y' + \frac{z_0}{2c}\Delta y'' - \frac{xyz_0}{bzc}\Delta c' + \frac{xyz_0}{bcz}\Delta c'' \quad \ldots\ldots (30)$$

$$\Delta y_{IL} = -\frac{(2x-b)yz_0}{2bcz}\Delta c' + \frac{(2x-b)yz_0}{2bcz}\Delta c'' \quad \ldots\ldots (31)$$

$$\Delta y_{IR} = -\frac{(2x+b)yz_0}{2bcz}\Delta c' + \frac{(2x+b)yz_0}{2bcz}\Delta c'' - \frac{z_0}{c}\Delta y' + \frac{z_0}{c}\Delta y'' \quad \ldots\ldots (32)$$

$$\Delta z_I = -\frac{(2x-b)z_0}{2bc}\Delta c' + \frac{(2x-b)z_0}{2bc}\Delta c'' \quad \ldots\ldots (33)$$

The remaining errors of the machine coordinates are found by adding the expressions (14) to (18) and (29) to (33):

(14) and (29):

$$\Delta x_I = +\frac{(2x-b)z}{2bc}\Delta x' - \frac{(2x+b)z}{2bc}\Delta x'' + $$
$$+\frac{(z-z_0)(2x+b)(2x-b)}{4bcz}\Delta c' - \frac{(z-z_0)(2x+b)(2x-b)}{4bcz}\Delta c'' \quad \ldots\ldots (34)$$

(15) and (30):

$$\Delta y_I = +\frac{yz}{bc}\Delta x' - \frac{yz}{bc}\Delta x'' - \frac{z_0+z}{2c}\Delta y' + \frac{z_0-z}{2c}\Delta y'' +$$
$$+\frac{xy(z-z_0)}{bcz}\Delta c' - \frac{xy(z-z_0)}{bcz}\Delta c'' \quad \ldots\ldots (35)$$

(16) and (31):

$$\Delta y_{IL} = +\frac{yz}{bc}\Delta x' - \frac{yz}{bc}\Delta x'' - \frac{z}{c}\Delta y' + \frac{y(z-z_0)(2x-b)}{2bcz}\Delta c' - \frac{y(z-z_0)(2x-b)}{2bcz}\Delta c'' \quad (36)$$

(17) and (32):

$$\Delta y_{IR} = +\frac{yz}{bc}\Delta x' - \frac{yz}{bc}\Delta x'' - \frac{z_0}{c}\Delta y' + \frac{z_0-z}{c}\Delta y'' + $$
$$+\frac{y(z-z_0)(2x+b)}{2bcz}\Delta c' - \frac{y(z-z_0)(2x+b)}{2bcz}\Delta c'' \quad \ldots\ldots\ldots (37)$$

(18) and (33):

$$\Delta z_I = +\frac{z^2}{bc}\Delta x' - \frac{z^2}{bc}\Delta x'' + \frac{z(2x+b)-z_0(2x-b)}{2bc}\Delta c' + \frac{(z_0-z)(2x-b)}{2bc}\Delta c'' \quad \ldots\ldots (38)$$

Taking account of:

$$\Delta z_I = -\Delta h_I \quad \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

in general matrix notation (34) to (38) is:

$$\begin{pmatrix} \Delta x_I \\ \Delta y_I \\ \Delta y_{IL} \\ \Delta y_{IR} \\ \Delta h_I \end{pmatrix} = (A_I^i) \begin{pmatrix} \Delta x' \\ \Delta x'' \\ \Delta y' \\ \Delta y'' \\ \Delta c' \\ \Delta c'' \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

with:

$$\begin{pmatrix}
+\frac{(2x-b)z}{2bc} & -\frac{(2x+b)z}{2bc} & 0 & 0 & +\frac{(z-z_0)(2x+b)(2x-b)}{4bcz} & -\frac{(z-z_0)(2x+b)(2x-b)}{4bcz} \\
+\frac{yz}{bc} & -\frac{yz}{bc} & -\frac{z_0+z}{2c} & +\frac{z_0-z}{2c} & +\frac{xy(z-z_0)}{bcz} & -\frac{xy(z-z_0)}{bcz} \\
+\frac{yz}{bc} & -\frac{yz}{bc} & -\frac{z}{c} & 0 & +\frac{y(z-z_0)(2x-b)}{2bcz} & -\frac{y(z-z_0)(2x-b)}{2bcz} \\
+\frac{yz}{bc} & -\frac{yz}{bc} & -\frac{z_0}{c} & +\frac{z_0-z}{c} & +\frac{y(z-z_0)(2x+b)}{2bcz} & -\frac{y(z-z_0)(2x+b)}{2bcz} \\
-\frac{z^2}{bc} & +\frac{z^2}{bc} & 0 & 0 & -\frac{z(2x+b)-z_0(2x-b)}{2bc} & +\frac{(z-z_0)(2x-b)}{2bc}
\end{pmatrix} \equiv (A_I^i)$$

(41)

Formulae suited to A8 measurements can be derived in the same way as to A7 measurements. The differential formulae which give the relation between machine coordinates and inner orientation elements are the same for both instruments, see (14) to (18).

The parallax caused by errors in inner orientation elements can be eliminated by the orientation elements $\varphi_2$, $\varphi_1$, $\kappa_2$ and $\kappa_1$. The parallax is in accordance with (20):

$$\Delta p_y = -\frac{z}{c}\Delta y' + \frac{z}{c}\Delta y'' - \frac{y}{c}\Delta c' + \frac{y}{c}\Delta c'' \quad \ldots \quad (42)$$

Comparing this formula with (31) of appendix 2 which gives the relation between $y$-parallax and the orientation elements of an A8:

$$\Delta p_y = -\frac{(2x-b)y}{2z}\Delta\varphi_2 + \frac{(2x+b)y}{2z}\Delta\varphi_1 + \frac{2x-b}{2}\Delta\varkappa_2 - \frac{2x+b}{2}\Delta\varkappa_1 \quad \ldots \quad (43)$$

From (42) and (43) we obtain for the mean level $z_0$:

$$\Delta\varkappa_2 = \frac{z_0}{bc}(\Delta y' - \Delta y'') \quad \ldots \quad (44)$$

$$\Delta\varkappa_1 = \frac{z_0}{bc}(\Delta y' - \Delta y'') \quad \ldots \quad (45)$$

$$\Delta\varphi_2 = -\frac{z_0}{bc}(\Delta c' - \Delta c'') \quad \ldots \quad (46)$$

$$\Delta\varphi_1 = -\frac{z_0}{bc}(\Delta c' - \Delta c'') \quad \ldots \quad (47)$$

In consequence of these corrections of the orientation elements the coordinates of the model points change, see (36) of appendix 2:

$$\begin{aligned}\Delta x_I = & -\frac{2x+b}{2b}\cdot\frac{(2x-b)^2+4z^2}{4z}\Delta\varphi_2 - \frac{(2x+b)y}{2b}\Delta\varkappa_2 + \\ & + \frac{2x-b}{2b}\cdot\frac{(2x+b)^2+4z^2}{4z}\Delta\varphi_1 - \frac{(2x-b)y}{2b}\Delta\varkappa_1 \quad \ldots \quad (48)\end{aligned}$$

$$\begin{aligned}\Delta y_I = & -\frac{xy(2x-b)+2yz^2}{2bz}\Delta\varphi_2 + \left(\frac{2x-b}{4} - \frac{y^2}{b}\right)\Delta\varkappa_2 \\ & + \frac{xy(2x+b)+2yz^2}{2bz}\Delta\varphi_1 + \left(\frac{2x+b}{4} + \frac{y^2}{b}\right)\Delta\varkappa_1 \quad \ldots \quad (49)\end{aligned}$$

$$\begin{aligned}\Delta y_{IL} = & -\frac{(2x-b)^2 y+4yz^2}{4bz}\Delta\varphi_2 - \frac{y^2}{b}\Delta\varkappa_2 + \\ & + \frac{(2x+b)(2x-b)y+4yz^2}{4bz}\Delta\varphi_1 + \left(\frac{2x+b}{2} + \frac{y^2}{b}\right)\Delta\varkappa_1 \quad \ldots \quad (50)\end{aligned}$$

$$\Delta y_{IR} = -\frac{(2x+b)(2x-b)y+4yz^2}{4bz}\Delta\varphi_2 + \left(\frac{2x-b}{2} - \frac{y^2}{b}\right)\Delta\varkappa_2 +$$

$$+\frac{(2x+b)^2 y+4yz^2}{4bz}\Delta\varphi_2 + \frac{y^2}{b}\Delta\varkappa_1 \quad \ldots\ldots\ldots\ldots (51)$$

$$\Delta z_I = -\frac{(2x-b)^2+4z^2}{4b}\Delta\varphi_2 - \frac{yz}{b}\Delta\varkappa_2 +$$

$$+\frac{(2x+b)^2+4z^2}{4b}\Delta\varphi_1 + \frac{yz}{b}\Delta\varkappa_1 \quad \ldots\ldots\ldots\ldots (52)$$

introducing (44) to (47):

$$\Delta x_I = -\frac{yz_0}{bc}\Delta y' + \frac{yz_0}{bc}\Delta y'' + \frac{(4z^2-4x^2+b^2)z_0}{4bcz}\Delta c' +$$

$$-\frac{(4z^2-4x^2+b^2)z_0}{4bcz}\Delta c'' \quad \ldots\ldots\ldots\ldots (53)$$

$$\Delta y_I = +\frac{xz_0}{bc}\Delta y' - \frac{xz_0}{bc}\Delta y'' - \frac{xyz_0}{bcz}\Delta c' + \frac{xyz_0}{bcz}\Delta c'' \quad \ldots\ldots\ldots\ldots (54)$$

$$\Delta y_{IL} = \frac{(2x+b)z_0}{2bc}\Delta y' - \frac{(2x+b)z_0}{2bc}\Delta y'' +$$

$$-\frac{(2x-b)yz_0}{2bcz}\Delta c' + \frac{(2x-b)yz_0}{2bcz}\Delta c'' \quad \ldots\ldots\ldots\ldots (55)$$

$$\Delta y_{IR} = \frac{(2x-b)z_0}{2bc}\Delta y' - \frac{(2x-b)z_0}{2bc}\Delta y'' +$$

$$-\frac{(2x+b)yz_0}{2bcz}\Delta c' + \frac{(2x+b)yz_0}{2bcz}\Delta c'' \quad \ldots\ldots\ldots\ldots (56)$$

$$\Delta z_I = -\frac{2xz_0}{bc}\Delta c' + \frac{2xz_0}{bc}\Delta c'' \quad \ldots\ldots\ldots\ldots (57)$$

From (14) to (18) and (53) to (57) we obtain by adding and taking account of (39):

$$\begin{pmatrix} \Delta x_I \\ \Delta y_I \\ \Delta y_{IL} \\ \Delta y_{IR} \\ \Delta h_I \end{pmatrix} \equiv (A_I^i) \begin{pmatrix} \Delta x' \\ \Delta x'' \\ \Delta y' \\ \Delta y'' \\ \Delta c' \\ \Delta c'' \end{pmatrix} \quad \ldots\ldots\ldots\ldots (58)$$

with:

$$\left.\begin{array}{lllll}
+\dfrac{(2x-b)z}{2bc} & -\dfrac{(2x+b)z}{2bc} & -\dfrac{yz_0}{bc} & +\dfrac{yz_0}{bc} & +\dfrac{(z-z_0)(2x+b)(2x-b)+4z_0z^2}{4bcz} \\
+\dfrac{yz}{bc} & -\dfrac{yz}{bc} & +\dfrac{2xz_0-bz}{2bc} & -\dfrac{2xz_0+bz}{2bc} & +\dfrac{xy(z-z_0)}{bcz} \\
+\dfrac{yz}{bc} & -\dfrac{yz}{bc} & +\dfrac{z_0(2x+b)-2bz}{2\,bc} & -\dfrac{z_0(2x+b)}{2bc} & +\dfrac{y(z-z_0)(2x-b)}{2bcz} \\
+\dfrac{yz}{bc} & -\dfrac{yz}{bc} & +\dfrac{z_0(2x-b)}{2bc} & -\dfrac{z_0(2x-b)+2bz}{2bc} & +\dfrac{y(z-z_0)(2x+b)}{2bcz} \\
-\dfrac{z^2}{bc} & +\dfrac{z^2}{bc} & 0 & 0 & -\dfrac{(2x+b)z-4xz_0}{2bc} \\
 & & & & -\dfrac{(z-z_0)(2x+b)(2x-b)+4z_0z^2}{4bcz} \\
 & & & & -\dfrac{xy(z-z_0)}{bcz} \\
 & & & & -\dfrac{y(z-z_0)(2x-b)}{2bcz} \\
 & & & & -\dfrac{y(z-z_0)(2x+b)}{2bcz} \\
 & & & & +\dfrac{(2x-b)z-4xz_0}{2bc}
\end{array}\right\} \equiv (A_l^i) \quad . \; . \; . \; . (59)$$

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